Seasonal Adjustment Procedures and Price Statistics

5

Prices

Bernd Schips
Winfried Stier

The CENSUS-X-11 Procedure
Theory, Assessment and Alternatives

BFS OFS UST

Swiss Federal
Statistical Office

Bern, 1995

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Published by: Swiss Federal Statistical Office (SFSO)
Information: Dieter Koch, SFSO, Tel. 031 322 28 43
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Obtainable from: Swiss Federal Statistical Office
CH-3003 Berne
Tel. 031 323 60 60
Order number: 156-0
Price: Sfr. 11.–
Series: Swiss Statistics
Field: 5 Prices
Original text: German
Translation: PD Dr. Bernard Michael Gilroy, University of St. Gallen
Prof. Dr. Winfried Stier, University of St. Gallen
The text in the figures has not been translated for reasons of time
Graphics/Layout: Authors
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ISBN: 3-303-05221-2

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0. Preliminary Remarks

In recent years, the CENSUS-X-11 seasonal adjustment procedure has achieved a wide degree of user popularity among various public bureaus of statistics, as well as by private research institutions in many countries. Such a positive resonance may be founded either upon acknowledged methodological advantages of the method or they may simply be an expression of a rather successful marketing strategy implemented by the founding fathers of the CENSUS-X-11 procedure.

For some time now, with regard to the methodological advantages and disadvantages of the CENSUS-X-11-method, a pertinent and growing literature is emerging generating an increasingly critical attitude from the methodological perspective of the CENSUS-X-11 procedure. Thus in Germany, for example, there were early attempts to develop an alternative methodology. The German Bureau of Statistics ("Statistische Bundesamt"), e.g. today uses the so-called BERLIN METHOD (Berliner Verfahren) [1], the study group of the leading German economic research institutions have for some time now been working on developing their own procedure [2]. However, despite these developments, the central bank of Germany still applies the CENSUS-X-11 method, albeit repeated convincing evidence from the scientific community that the procedure has various methodological shortcomings [3]. Due to these inadequacies, especially for the most recent values for example, Canada now applies a special version of the CENSUS-X-11 procedure [4] and most recently, the CENSUS-Bureau of the United States is now working on a modified version X-12 [5].

Since the methodological misgivings of the CENSUS-X-11 procedure are well known and commonly acknowledged, it must certainly be the case that the wide-spread usage of the method, to some extent, is a result of successful marketing by it's developers. The CENSUS-X-11 procedure was one of the first available, largely portable programms for seasonal adjustment. Consequently, it filled a market niche and found rapid approval and approbation. Thus, the impression was created that under the implementation of one and the same programm a direct comparison of seasonally adjusted data was possible. Even today this notion is still partly held, although the numerous parameters necessary to implement the procedure and the now common generally applied modifications lead to compulsory and sometimes very large methodologically contingent differences in derived results.

Therefore the purpose of this study is to analyse the principal methodological objections against using the CENSUS-X-11 procedure and to demonstrate their effects. Our arguments will be exemplified using Swiss price series (wholesale - and consumer price indices). Finally, we offer recommendations for methodologically acceptable adjustment procedures and a discussion on how to handle adjusted data.

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I. How does one identify seasonality?

1. Introduction

It is well known that in many semi-annual economic time series, e.g. monthly-, quarterly-, half-year-time series, an seasonal influence can be found. It is, however, less well-known, by which means one can *generally* identify, whether or not seasonality is present in a concrete time series, the exception being those time series in which seasonality can be clearly identified by simple graphical observation. It is not uncommon, however, to have time series data that do not permit such a simple recognition of seasonal influences. Most of the especially interesting data on price series belongs to this category. Examination of such time series therefore requires the aid of special methods to identify whether a seasonal component exists.

The following presents and discusses the important practical aspects of these methods of analysis. The approach taken is to hold the analysis of the more formal-mathematical details of the various methods to a necessary minimum and to highlight the important ideas and fundamental relationships graphically [1].

2. Stochastic Processes

2.1 Fundamentals

A characteristic of modern day time series analysis is that the theoretical framework applied in time series models is the *Theory of Stochastic Processes*. Consequently, time series models are *stochastic* models. The implication of this is that concrete time series are interpreted as realizations of stochastic processes.

A *stochastic process* is defined as a set of random variables $\{X_t \mid t \in T\}$ which are dependent upon an index set T constituting the parameter space of the process. In time series analysis T always means a set of time points. Two different cases must be considered. In case one, T represents an intervall in $\mathbb{R}$, and T is designated as a *continuous parameter space*. In case two, T is either a finite set of real numbers or T contains at least in some closed interval only a finite amount of numbers, e.g. $T = \{0,1,2,...\} := \mathbb{N}$ or $T = \{0, \pm1, \pm2, ...\} := \mathbf{Z}$. T then represents a *discrete parameter space*. Given that T is finite, then $\{X_t \mid t \in T\}$ is designated as a *finite* stochastic process. It is assumed here, that T is always discrete. This case is especially important, since data in practice are commonly in discrete form. With regard to economic data, this is always the case.

Stochastic processes may also be characterized by their moments, just as is the case for ordinary random variables. Generally, however, it is appropriate here to speak of *moment functions*, since the moments of stochastic processes are fundamentally dependent upon the parameter $t \in T$. The expression

$$\mu_t := E[X_t]$$

is called the *expected value function (or mean function)* and

$$\sigma_t^2 := Var[X_t]$$

represents the *variance function* of the stochastic process X_t. The *covariance funtion* is defined as

$$Cov(X_s, X_t) = \gamma(s,t) := E[(X_s - \mu_s)(X_t - \mu_t)],\ s, t \in T$$

and the respective *correlation function* by

$$Cor(X_s, X_t) = \rho(s,t) := \frac{Cov(X_s, X_t)}{[Var(X_s)Var(X_t)]^{1/2}} .$$

2.2 *Stationary Stochastic Processes*

In practical applications, an important class of stochastic processes are the *stationary processes* and within this class especially the *weak stationary processes*. A stochastic process is called *strictly stationary* given that it's distribution function may be characterized by

$$F(x_1,x_2,..,x_n; t_1,t_2,..,t_n) = F(x_1,x_2,...,x_n; t_1+\tau,t_2+\tau,...,t_n+\tau)$$

for all $n \in \mathbb{N}$, all n-tupel $(t_1,t_2,...,t_n) \in T^n$ and for all τ for which $(t_1+\tau, t_2+\tau,...,t_n+\tau)$ lies again in T^n.

Consequently, the distribution of n random variables $X_1, X_2,...,X_n$ given the points of time $t_1,t_2,...,t_n$ and $t_1+\tau$, $t_2+\tau,...,t_n+\tau$ are equivalent. Strictly stationary processes are thus invariant to shifts in the observation time points, i.e. they are *time-translation invariant*. For such a process it follows that both the expected value function as well as the variance function are time independent, that is to say constant. However, we have

$$F(x_1,x_2; t_1,t_2) = F(x_1,x_2; t_2-t_1) ,$$

which implies that the *second order* distribution function (i.e. for n=2) depends (only) on the time difference t_2-t_1. It follows then, that also the covariance and the correlation functions of a strictly stationary process depend (only) upon the time difference.

For practical time series analysis the assumption of strict stationarity is usually too limiting. Sufficient is the assumption of *weak stationarity*, for which the following holds:

$$\mu_t = \text{const.} ,$$

$$\sigma_t^2 = \text{const.} ,$$

$$\gamma(s,t) = \gamma(t-s)$$

and

$$\rho(s,t) = \rho(t-s) .$$

For weak stationary processes one often finds in the literature the terminology *stationary in a wide sense*, *covariance stationary* or simply *stationary*. In the following, therefore, our usage of the term "stationary" will imply throughout weak stationarity. Without limitations it may be generally be assumed that $\mu_t = 0$, since this may always be obtained by subtracting a constant from the process. This simplifies many of the formal expressions one obtains.

Given weakly stationary processes, one is so to speak only interested in the moment functions up to the second order, all higher moments are left out of consideration. Strictly stationary processes are also always weakly stationary, the reversed situtation is valid only for special cases.

The simplest stationary process may be characterized by pairwise uncorrelated random variables ε_t and an expected value of zero with variance σ^2. It follows that

$$\mu_t = 0 ,$$

$$\sigma_t^2 = \sigma^2$$

and

$$\gamma_\varepsilon(h) = \begin{cases} \sigma^2, & h = 0 \\ 0 , & h \neq 0 \end{cases}$$

respectively,

$$\rho_\varepsilon(h) = \begin{cases} 1 , & h = 0 \\ 0 , & h \neq 0 . \end{cases}$$

The above process is called *white noise.*

A more complicated process is, for example, given by

$$(X_t - \mu) = \phi\,(X_{t-1} - \mu) + \varepsilon_t\ ,$$

whereby ε_t represents white noise. Such a process is called a *first-order autoregressive process.* As it's name suggests, this process is characterized by the fact that the process regresses in each time point on itself, whereby the time-lag consists of one time unit, μ designates the expected value of X_t and ϕ the *process parameter.* This process is not stationary for arbitrary process parameters ϕ, only for those characterized by a magnitude smaller than one.

The moment functions of a AR(1)-process are as follows:

$$\mu_t = 0,\quad \sigma_t^2 = \frac{\sigma_\varepsilon^2}{1-\phi^2}\ ,$$

$$\gamma(h) = \left[\frac{\sigma_\varepsilon^2}{1-\phi^2}\right]\phi^h,\quad h = 0,1,2,\ldots\ ,$$

and

$$\rho(h) = \phi^h,\quad h = 0,1,2,\ldots$$

Generally, a finite-order *autoregressive process of order p* or briefly, *AR(p)-Process*, has the form

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t$$

with $\phi_p \neq 0$, whereby for simplifying reasons (without limiting, however, it's general validity) it is assumed, that the expected value of X_t is equal to zero. Also, in this case, the process parameters may not be chosen arbitrarily if the AP(p)-process is to be stationary. Due to the complexity of the stationarity conditions, a detailed discussion of their properties is beyond the scope of this study. For the same reason, we also forgo presenting a detailed analysis of the moment functions.

The process

$$X_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q}\ ,$$

whereby ε_t represents white noise, is called a (finite) Moving-Average-Process of order q - or briefly a MA(q)-Process - with the process parameter $\theta_0 = 1$, $\theta_1,\ldots,\theta_q$ with $\theta_q \neq 0$. MA-Processes are nothing more than weighted averages of uncorrelated random variables, whereby the sum of the weights 1, $\theta_1,\ldots,\theta_q$ need not sum to unity. Obviously, MA-processes are stationary for arbitrary parameters. For the following MA(1)-process

$$X_t = \varepsilon_t + \theta\varepsilon_{t-1}\ ,$$

for example, the expected value and variance functions are

$$\begin{aligned} E(X_t) &= E(\varepsilon_t) + \theta E(\varepsilon_{t-1}) = 0 \\ Var(X_t) &= Var(\varepsilon_t) + \theta^2 Var(\varepsilon_{t-1}) \\ &= \sigma_\varepsilon^2(1 + \theta^2) \end{aligned}$$

Also, the autocovariance, respectively, autocorrelation functions may be stated as

$$\begin{aligned} \gamma(1) &= E\,[(\varepsilon_t + \theta\varepsilon_{t-1})(\varepsilon_{t-1} + \theta\varepsilon_{t-2})] \\ &= \theta E(\varepsilon_{t-1}^2) \\ &= \theta\sigma_\varepsilon^2 \\ \gamma(h) &= 0 \text{ for } h \geq 2 \end{aligned}$$

and

$$\rho(1) = \frac{\theta}{1+\theta^2}$$

$$\rho(h) = 0 \text{ for } h \geq 2\ .$$

It is also possible to combine both process types. This process is referred to as an ARMA-Process of order (p,q) - or simply as an ARMA(p,q) process, defined as

$$X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \varepsilon_t + \theta_1\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q}.$$

We also omit giving a detailed discussion of the stationarity conditions and respective moment functions for this class of processes.

3. Fundamentals of Spectral Analysis

3.1 Preliminary Remarks

The previous discussion may be characterized as being exclusively formulated in the *time-domain.* For example, the moment functions are generally defined as functions of time. There is, however, another alternative way of looking at the situation which permits an illustration of a stationary process in a simple vivid manner. Hereby, one observes the so-called *frequency domain.* At first glance, it might appear to be the case that such an approach is considerably more complicated than similar considerations applying normal time-domains. It will, however, be demonstrated simply in the following examples and graphs, that analysis using frequency domains and if necessary a combination of both frequency- and time-domains may be a preferable alternative to a pure time-domain analysis.

Time domain and frequency domain may not be considered being isolated from one another. Rather, by applying a

special mathematical transformation a conversion of both domains may be obtained. Both domains exhibit consequently a dual nature. It is beyond the scope of this study to discuss explicitly this transformation. Instead, the duality will be demonstrated applying a simple example.

For the purpose of illustration, assume now a (discrete) sinus-wave given a period of T = 12 time units (TU), e.g. 12 months. This means that the observed time series X_t may be described in the following manner:

$$X_t = \sin\lambda t \ , \ t = 1,2,... \ ,$$

with

$$\lambda = 2\pi/12 \ .$$

Examining the time domain one obtains the following pattern (cf. Figure 1):

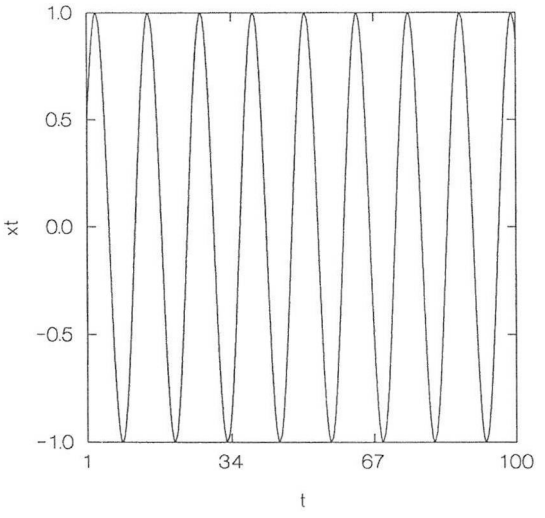


Figure 1

Hereby, λ is the frequency of the oscillations expressed in radians (rad), which amounts here to 0.5236. Their frequency in number of cycles per time unit is f = 1/12. In general, the relationship between both of these frequency concepts may be given as

$$\lambda = 2\pi f \ ,$$

whereby $0 \leq f \leq 0.5$. That f may maximally take on a value of 0.5, is due to the fact that given a discrete series no periods smaller than 2 time units can be observed. For example, observing a monthly series it is not possible to regard an oscillation of say a time period of three weeks.

If one speaks of sinus-oscillations possessing a period of 12 TU, then we are thinking in the time domain. One may just as well, however, argue that they exhibit a frequency of f = 1/12, i.e. 1/12 of the cycle "passes by" so to say per TU. Naturally, it is possible to express this in radians λ. However, such an interpretation is generally less clear. Consequently, in the following, as a rule, frequencies will be expressed as a number of cycles/TU.

The question immediately arises naturally in the above simple example, what practical advantages are obtainable given one expresses the above mentioned series in the frequency domain instead of the usual time domain. Given the present example, there does not appear to be any apparent advantage. However, the situation changes drastically when one studies more complicated series. In such cases, the analysis in the frequency domain offers at least two advantages. First of all, it give us information about the "frequency content" of such series and secondly, it tells us what the effects are of transforming series (for example, through smoothing, taking first differences or general filtering) on the frequency content. Usually, the frequency content of a series is expressed by its spectrum.

3.2 Spectra of Stationary Processes

To begin with we consider a stationary process X_t with a respective covariance function $\gamma(h)$. The *spectrum* (also referred to as "power spectral density function") of X_t is defined by

$$S(f) = \sum_{h=-\infty}^{\infty} \gamma_h e^{i2\pi fh}$$

$$= \gamma_0 + 2 \sum_{h=1}^{\infty} \gamma_h \cos 2\pi fh \ .$$

The spectrum is defined as a *Fourier transform* of the covariance function. Substituting the correlation function in the transformation instead of the covariance function one obtains the so-called *spectral density*. It is derived given S(f) by dividing through with the variance of the X_t.

Both the spectrum and spectral density are even, continuous, periodical and non-negative functions of frequency f. The integral over S(f), i.e. the area under the spectrum, is equal to the variance of the process, whereas the integral over the spectral density function is equal to one. The spectral density function is thus formally similar to the density function of a random variable, which explains the connotation "spectral density function".

The spectrum of a process reflects the division of the process variance over the frequencies resp. frequency bands: given a specific frequency band of a respective process, a component is even more important, the greater the "spectral mass" is in its corresponding frequency band. Generally, for the frequency band $[f_1, f_2]$ the integral

$$\int_{f_1}^{f_2} S(f)\,df$$

denotes the share of the process variance given oscillation components with frequencies $f \in [f_1 , f_2]$.

Let us now, for purposes of illustration, observe some simple stochastic processes. Given the above mentioned white noise, for example,

$$S(f) = \sigma^2 ,$$

i.e. the spectrum of a white noise process is constant. No frequency dominates the process, rather all frequencies have the same contributing effect upon the variance of the process (cf. Figure 2 for $\sigma^2 = 1$):

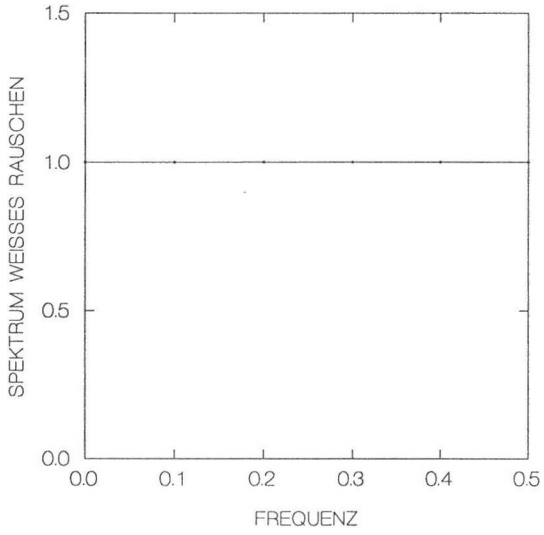


Figure 2

An AR(1)-Process is characterized by the following spectrum:

$$S(f) = \frac{\sigma^2}{1+\phi^2-2\phi\cos 2\pi f} .$$

The pattern of the above spectrum depends thus upon the process parameter ϕ. For positive ϕ, the spectral mass is largely concentrated in the "low frequency interval" (i.e. mainly in the proximity of the frequency $f = 0$). Whereas for negative ϕ, a concentration in the "high frequency interval" occurs (i.e. approximately in the vicinity of $f = 0.5$). The following Figures 3 and 4 display the spectra of AR(1)-processes given $\phi = 0.7$ and $\phi = -0.7$ and $\sigma^2 = 0.1$, respectively.

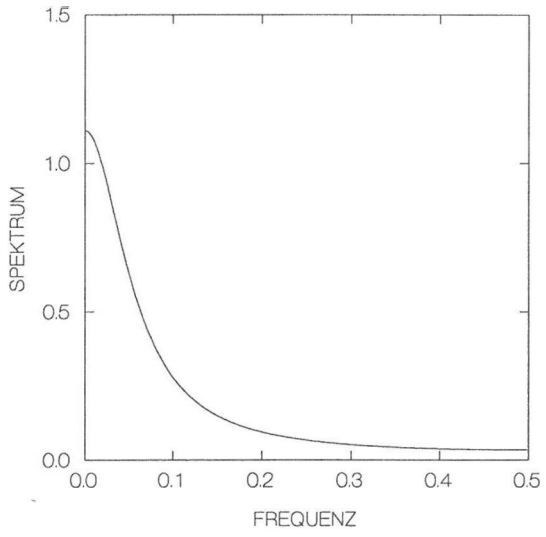


Figure 3

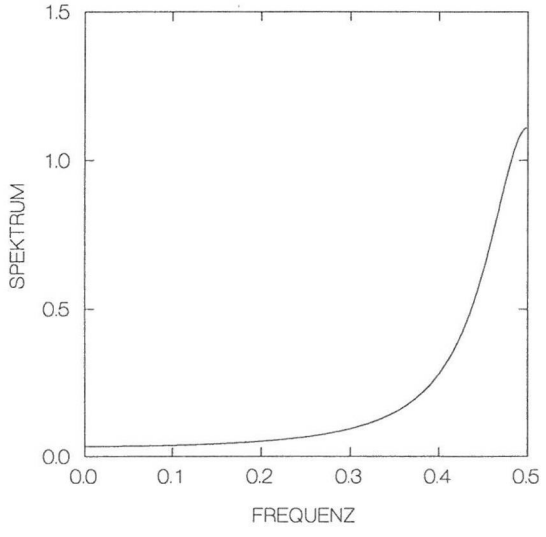


Figure 4

Figures 5 and 6 display single realisations of 100 observations of the above mentioned processes.

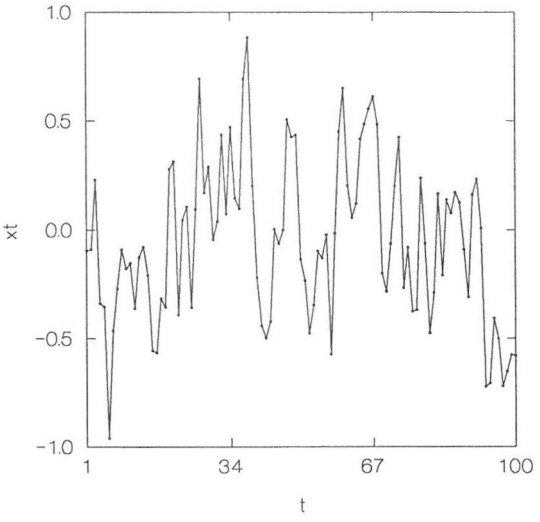


Figure 5

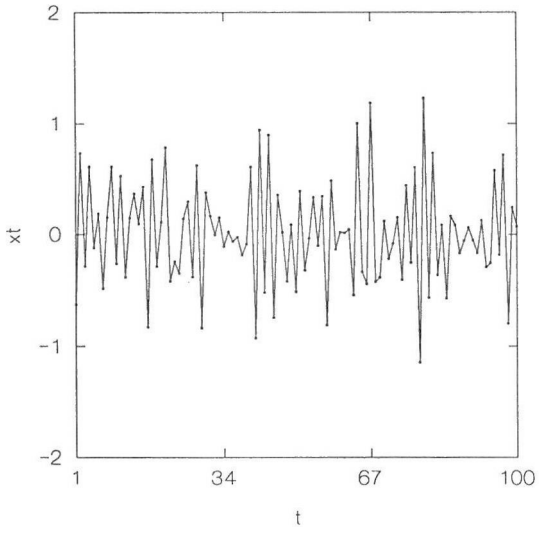


Figure 6

Looking first at the time series presented in Figure 5, one observes a relatively smooth path (please note the scales). On the other hand, Figure 6 is characterized by very erratic up and down movements. The paths of these processes are naturally not due to pure coincidence, rather it is much more the case that they are to be always expected given the postulated spectra. The dominating low frequency domain due to a positive ϕ leads to relatively slow oscillations, whereas in the high frequency domain (especially for the frequency f = 0.5) a large negative ϕ induces oscillations. Consequently, the spectrum contains information regarding the *pattern* of the process realisations, i.e. of the time series. The above mentioned fact is even more pronounced in the following example, the spectrum of a AR(2)-process.

The equation for an AR(2)-process may be characterized as follows,

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t ,$$

with the corresponding spectrum

$$S(f) = \frac{\sigma^2}{1 + \phi_1^2 + \phi_2^2 + 2(\phi_1\phi_2 - \phi_1)\cos 2\pi f - 2\phi_2 \cos 4\pi f} .$$

Assuming the parameter constellation $\phi_1 = 1.6$, $\phi_2 = -0.8$ and $\sigma^2 = 0.1$, the spectrum exhibits the following path (cf. Figure 7):

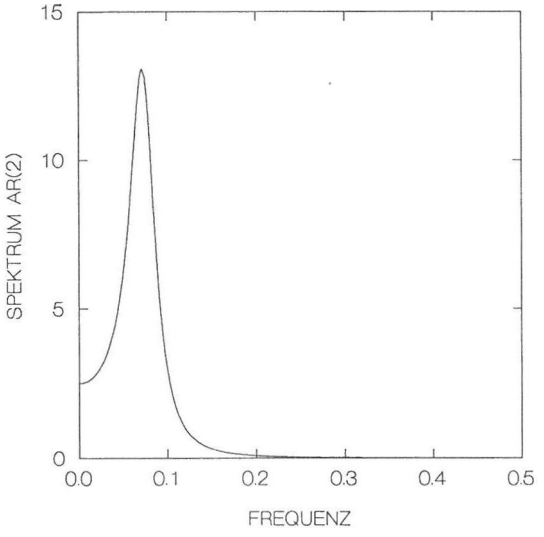


Figure 7

This spectrum is especially characterized by a very accentuated peak at the frequency f = 0.074. That is, the spectral mass is highly concentrated "around" this frequency,leaving all other frequencies to be of relatively less importance.

If one now chooses the parameters $\phi_1 = 1.6$, $\phi_2 = -0.99$ and $\sigma^2 = 0.1$, then the following spectrum (see Figure 8) arises:

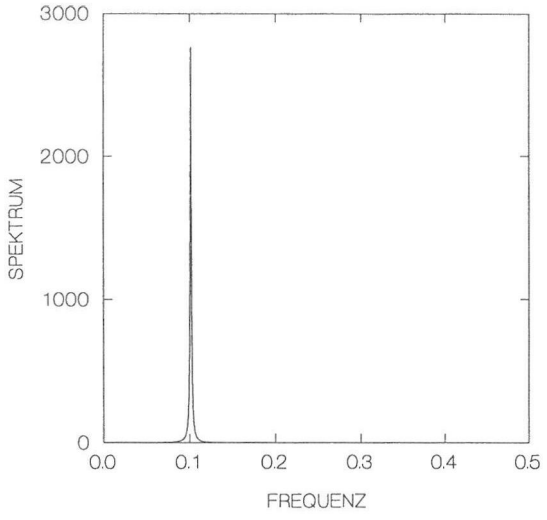


Figure 8

As readably observable, this spectrum differentiates itself from the previous one in the fact that practically it's entire spectral mass is concentrated around the frequncy f = 0.10 (please take note of the different scaling of the Figures 7 and 8). Stating it in a more simple manner, one could say that the spectrum for all frequencies is zero, with the exception of the frequency f = 0.10, at which it takes on a very large magnitude (circa 2500).

The following Figures 9 and 10 illustrate a realisation of both the above AR(2)-processes.

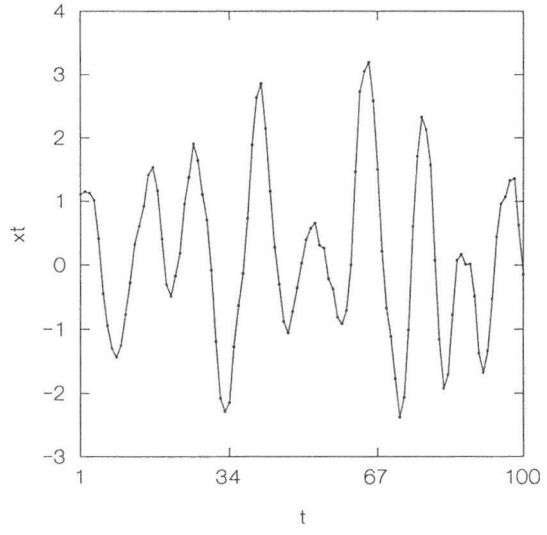


Figure 9

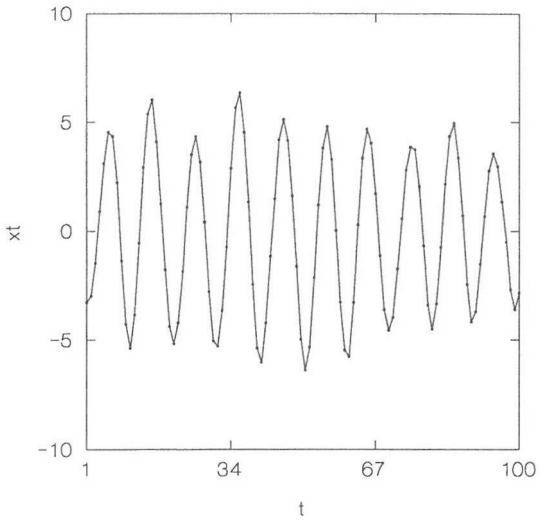


Figure 10

Observing the first process, the realisation shows a quasi-periodical path with a period of some 13 TU. The second process, however, demonstrates an almost exact periodical development with a period of 10 TU.

These examples illustrate that accentuated peaks in the spectrum may induce quasi-periodical movements (oscillations) in time series. The "narrower" and stronger accentuated such peaks are in comparison to other values of the spectrum, the more regular are paths of the processes.

The above presented characteristics of spectra are of importance when considering the seasonal components of time series, because given seasonal movements one is basically dealing with quasi-periodical processes. As such, it is to be expected that time series, given their spectrum peaks are concentrated "around" certain frequencies, may generate a seasonal component. Such frequencies are termed "*seasonal frequencies*". For monthly time series these are the frequencies 1/12, 2/12, 3/12, 4/12, 5/12 and 6/12, whereby usually as a rule the very first frequency dominates. The corresponding frequencies exhibiting 2/12 - 6/12 oscillations are commonly designated as "*harmonics*" of the fundamental frequency 1/12. Contingent upon the specific situation at hand, these harmonics may be more or less observable, or even be absent. Given quarterly series, 1/4 and 1/2 are the respective seasonal frequencies.

3.3 Estimating the Spectrum

Up to now it was assumed, that the generating process of the time series is known. As demonstrated in the above section, the spectrum of a known stationary process may be *calculated*. Unfortunately, given more practical applications one is in a more unfavorable situation. The generating process is unknown. Observable is only *one* process realisation, that is a given time series, which is also always finite and often even rather short. The time series values x_1,

$x_2,\ldots, x_T$ are then perceived to be realisations of the random variables $X_1,X_2,\ldots,X_T$.

Consequently, the problem arises that the spectrum must be *estimated* with the aid of such time series. One natural estimation method is given by the definition of the spectrum introduced in section 3.2 above, whereby one substitutes the unknown covariance function with a corresponding *estimated* covariance function. Such an estimator may be defined as

$$\hat{\gamma}_h = \frac{1}{T}\sum_{t=1}^{T-h}(X_t - \bar{X}_T)(X_{t+h} - \bar{X}_T), \quad h = 0,1,2,\ldots,$$

whereby

$$\bar{X}_T = \frac{1}{T}\sum_{t=1}^{T} X_t$$

represents an estimator for the expected value μ and

$$\hat{\gamma}_0 = \frac{1}{T}\sum_{t=1}^{T}(X_t - \bar{X}_T)^2$$

defines an estimator for the variance σ^2 of the process.

Since the unknown covariance function can not be estimated for an infinite number of lags, the above stated infinite sum must be replaced by a finite one, i.e. summed to a maximal lag M, which must be specifically determined. Following the above procedure, one obtains the following estimator for the unknown spectrum

$$\hat{S}(f) = \hat{\gamma}_0 + 2\sum_{h=1}^{M}\hat{\gamma}_h \cos 2\pi f h \quad .$$

It must, however, be observed that the theoretical spectrum is a continuous function of the frequency f, whereas an estimation is naturally only based upon a finite number of frequency points (e.g. for the frequencies f = 0, 0.005, 0.01, 0.015, ..., 0.495, 0.5).

Such an estimator, commonly referred to as *periodogram*, exhibits a *serious* disadvantage; though it is (asymptotically) unbiased, it is *not consistent*. This implies that the variance of the periodgram does not shrink to zero as the sample population becomes infinitely large. Thus, in principle, the periodgram may not be adequately applied as an estimator of the spectrum. Furthermore, it may not be applied to examine the "periodicity" of time series. However, it is possible to obtain a consistent estimator given a certain modification of the periodogram.

It can be shown that a consistent estimator of the spectrum may be derived by multiplication of the estimated covariances in the periodogram with a sequence of weights w_h. Such a sequence is characterized by the fact that after a specific lag h = M (the so-called *truncation point*) it takes on the value zero. It is commonly given the name *lag-window*. It is possible to construct lag-windows in different ways. A simple example of a lag-window is the BARTLETT-window with the weights

$$w_h = \begin{cases} 1 - \frac{h}{M} & \text{for } h \le M \\ 0 & \text{for } h > M \end{cases} \quad .$$

In practical applications of spectral estimation, the PARZEN-window is one of the most frequently used. We adhere to this tradition in the following spectral estimations. The weights applied are

$$w_h = \begin{cases} 1 - 6(\frac{h}{M})^2 + 6(\frac{h}{M})^3 & \text{for } h \le \frac{M}{2} \\ 2(1 - \frac{h}{M})^3 & \text{for } \frac{M}{2} < h \le M \\ 0 & \text{for } h > M \end{cases} .$$

The various windows have specific advantages and disadvantages, which will not be discussed here.

The respective estimator for the spectrum is thus

$$\hat{S}(f) = \hat{\gamma}_0 + 2\sum_{h=1}^{M} w_h \hat{\gamma}_h \cos 2\pi f h \ ,$$

whereby w_h designates the PARZEN-window. This estimator is asymptotically unbiased and consistent.

Estimators using a lag-window for the covariance function are called *indirect* spectral estimators. Looking at their properties in the frequency domain, it may be demonstrated that an estimation of a specific frequency point $f = f_0$ is basically the same thing as an average taken over the spectral values of f_0 and the spectral values of "neighboring" frequencies of f_0. Such averaging- or smoothing properties of spectral estimators may lead to "leakage" problems, such that an estimation at f_0 may be heavily influenced or "falsified" by very large (or very small) neighboring spectral values. The spectral mass of neighboring frequency points, so to speak, "leaks" through and may result in non-informative spectra. Given these circumstances, it might be necessary to "filter out" spectral mass before estimating a spectrum. This problem plays an important role in examining the seasonality of economic time series (cf. the discussion in the following section 3.4).

With regard to practical applications, a special problem arises: how should the *truncation point M* be determined? It can be shown that the *resolution* of the spectral estimator improves increasingly with a larger M, i.e. closely neighboring peaks of the theoretical spectrum are also evident in the estimated spectrum and are not simply observed as one peak. In other words, the bias of the estimation is all the more smaller the larger M is. This bias is always present given finite samples and leads to the tendency of underestimation of "peaks" in the spectrum and the tendency to overestimate "troughs". However, the variance of the estimator is all the more larger, the larger the *truncation point* chosen. Thus, the estimation becomes instabile as M

increases. Accordingly, an antinomy exists between the desired estimation properties *bias reduction* and *variance reduction*. As experience shows, by choosing M between T/5 and T/3 relatively acceptable results may usually be obtained, offering a good compromising solution taking regard of the above mentioned diverging properties.

Figure 11 illustrates the estimated spectrum of Figure 10 once again, whereby now a PARZEN-window with M = 20 has been chosen.

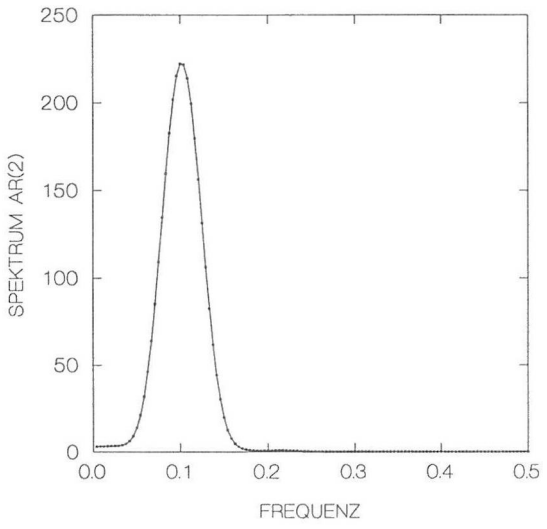


Figure 11

3.4 *Detecting Seasonal Movements Applying Spectral Analysis*

Now that the stochastic and theoretical background of spectral analysis has been briefly discussed, we continue our discussion by demonstrating how implementation of spectral analysis can be used to detect seasonality in time series. In so doing, for special reasons that will become evident during the course of presentation, we first of all use simulated time series whose "information content" for both the time - and frequency domains is known.

We start with the very simple sinus-wave, characterized by a period of 12 TU of length T = 100 as presented above in Figure 1.

This time series is to a certain extent a characteristic model of a seasonal figure. It is exactly periodical, completely deterministic and contains no further components. Given *infinite length*, however, based upon specific mathematical reasoning there exists no spectrum for this series as commonly defined. Despite this fact, it is possible by introducing a so-called *delta-function* to once again define a spectrum. The delta-function is a function which takes on the value zero everywhere and for one single point within the interval for which it is defined becomes infinitely large. Given a sinus-wave of infinite length, the corresponding delta-function is a function taking on the value of zero for all frequencies with the exception of the frequency f = 1/12, at which point the function exhibits an infinitely large value. The entire spectral mass is concentrated in this point. We avoid presenting a more detailed discussion of these aspects here, since they are not further relevant for our purpose at hand. On the other hand, postulating a *finite* sinus function, the spectral mass remains concentrated in a single frequency point, however, not exclusively. Applying the methods discussed above, it is possible to estimate the spectrum of such a wave. In doing so, one should expect to obtain an estimated spectrum whose spectral mass is symmetrically distributed around the frequency 1/12. This is illustrated below in Figure 12.

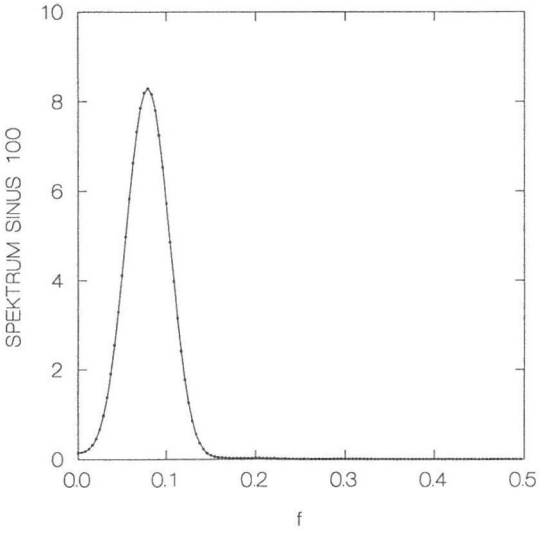


Figure 12

Examining the above spectrum at the frequency 1/12 more closely, it may be clearly observed, that the "peak" is located at 12 TU (cf. Figure 13):

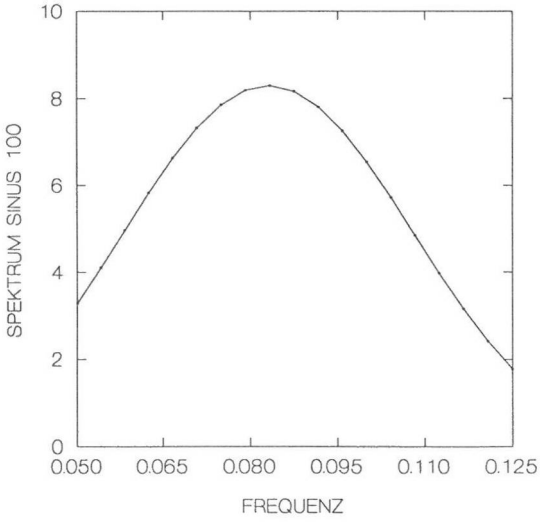


Figure 13

If the above sinus signal is successively elongated and the spectrum estimated, then it should be expected that the ordinate of the spectrum at f = 1/12 becomes increasingly larger and the spectral mass is concentrated more and more around this frequency. Figure 14 illustrates the estimated spectra for the values T = 100, 200, 500, 1000, 5000.

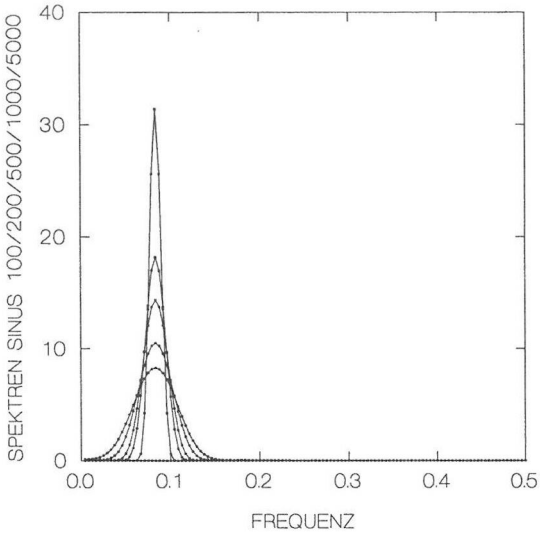


Figure 14

As is evident, these spectra converge to the above mentioned delta-function.

In the following, let us now introduce an additional component of white noise into our analysis of the observed simple time series. We differentiate among three different cases at hand characterized by the variances σ^2 = 0.1, 1.0 and 2.0. These three noise-components are illustrated below in Figures 15-17.

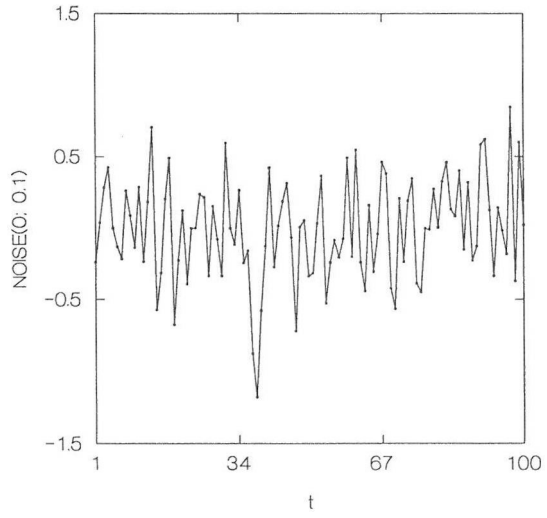


Figure 15

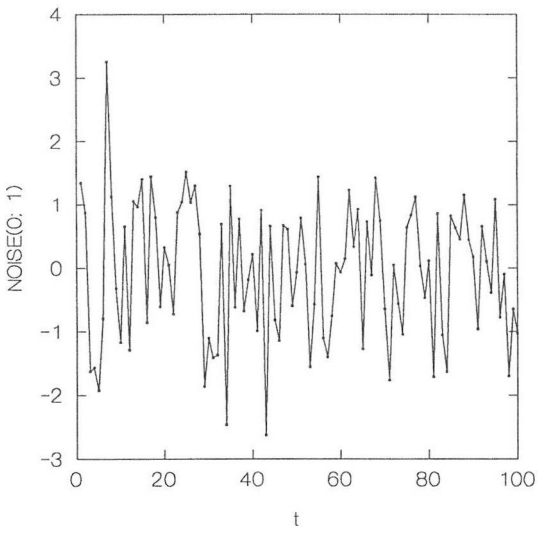


Figure 16

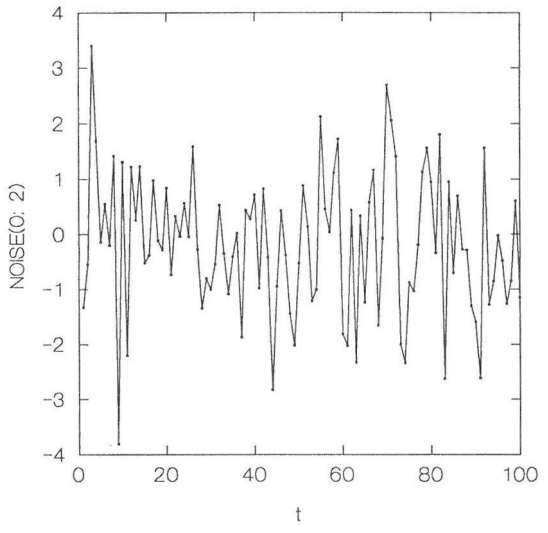


Figure 17

Now adding these noise-components to the above mentioned sinus-signal, the following series (cf. Figures 18, 19 and 20) are obtained:

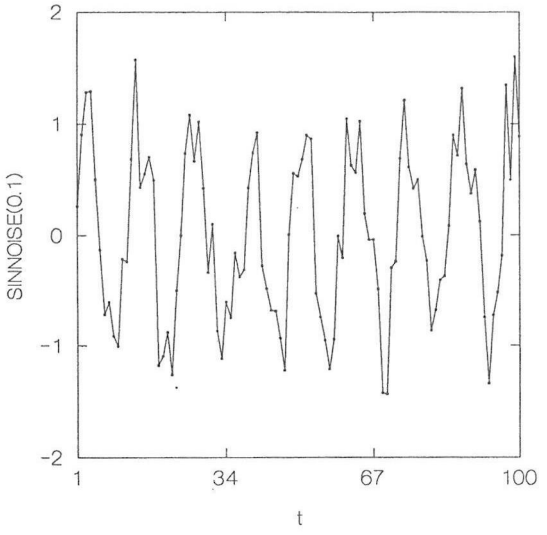


Figure 18

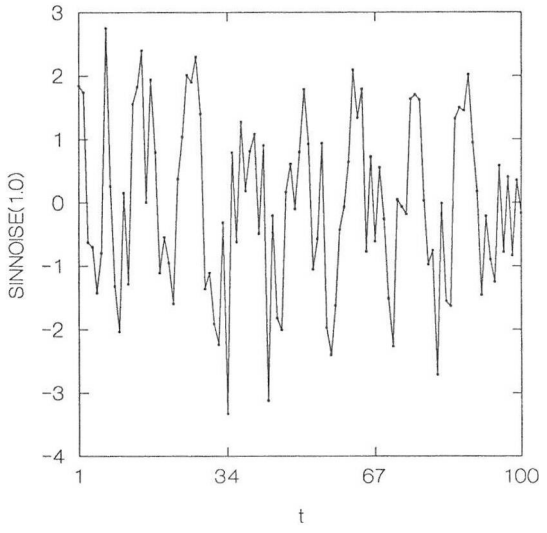


Figure 19

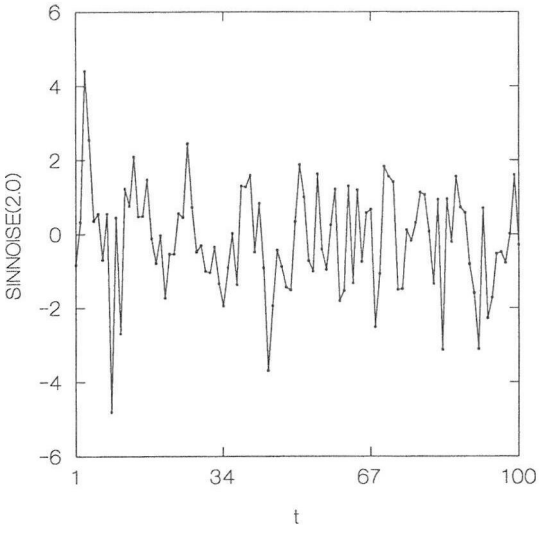


Figure 20

Whereas in Figure 18 the sinusoidal seasonal component is still clearly recognizable despite the noise interference, the larger variance of the noise-component in Figure 19 largely wipes out any observable regular seasonal pattern. Finally, in Figure 20, no aspects of seasonality are further observable at all.

Estimating the spectra of these three series, one obtains the following results (cf. Figures 21-23):

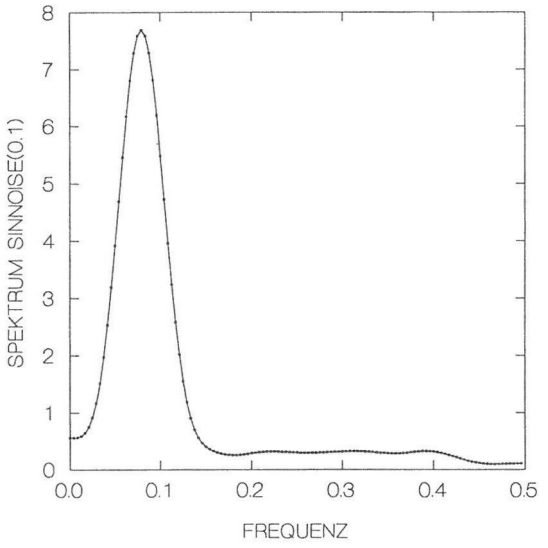


Figure 21

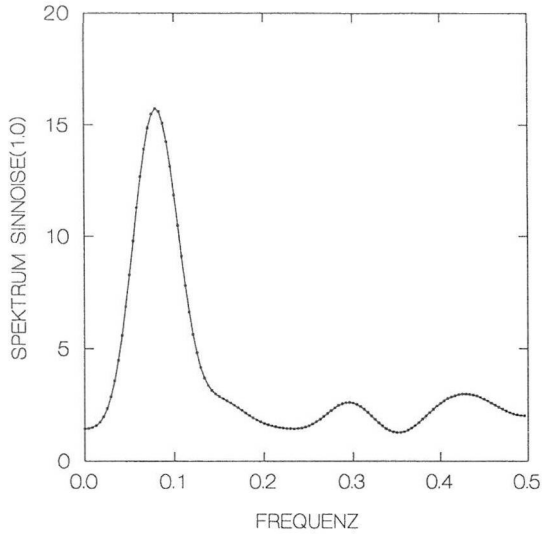


Figure 22

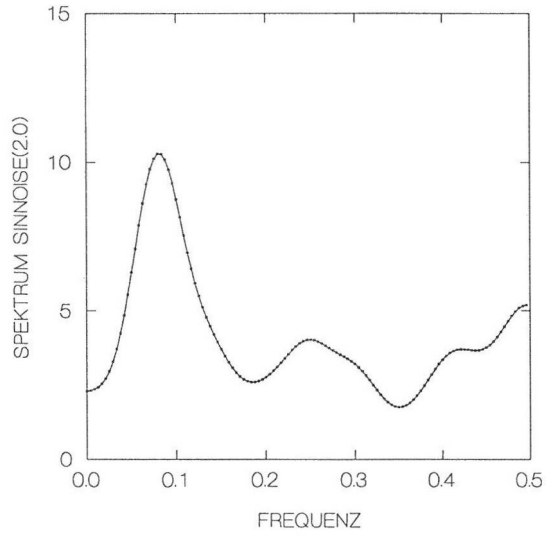


Figure 23

In all three cases, the respective estimated spectra unequivocally display a seasonal component. In addition to this, they also exhibit spectral mass at higher frequencies. This spectral mass increases the higher the level of the noise-variance identified. At the same time, however, in the low frequency domain and at the seasonal frequency 1/12, the spectral mass increases. The latter is due to the fact that the spectrum is constructed from a series consisting of two components that are combined additively from the partial spectra of the sinus wave and the noise component. Figure 24 presents these three spectra in one graph, illustrating the above circumstances.

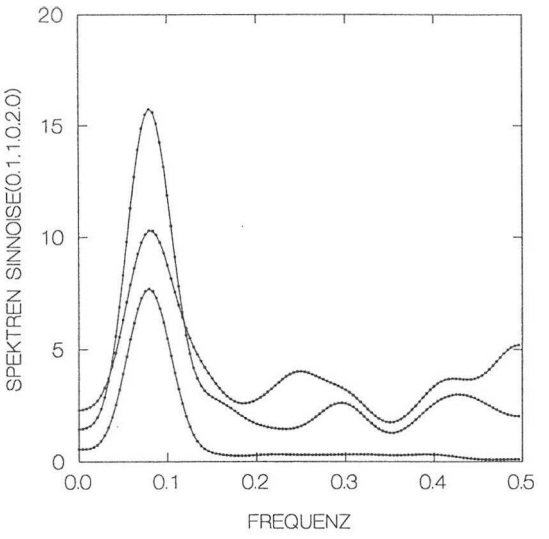


Figure 24

As is well known, economic time series commonly exhibit a (usually positive) trend. The next step then is to examine the effects of such a trend in addition to the above mentioned components when applying spectral analysis. For simplicity, consider the following series consisting of two components, the previously discussed sinusoidal seasonal figure and a linear trend

$$X_t = 2 + 0.9t + \sin(2\pi/12)t \ , \ t = 1,...,100 \ .$$

Looking at Figure 25, despite the dominating trend development given, the seasonal component is still very well recognizable.

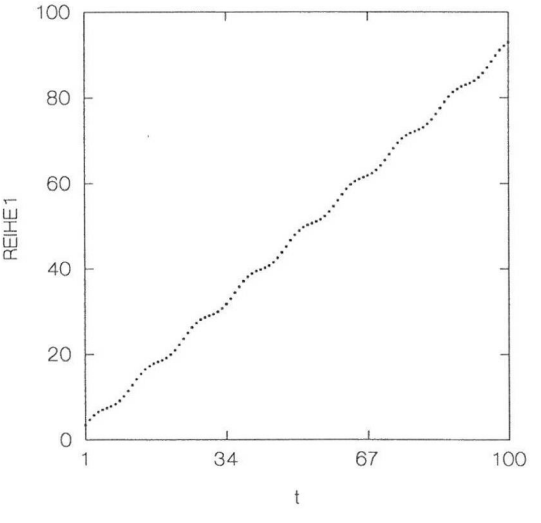


Figure 25

It is even more astonishing that the estimated spectrum exhibits the following path (see Figure 26):

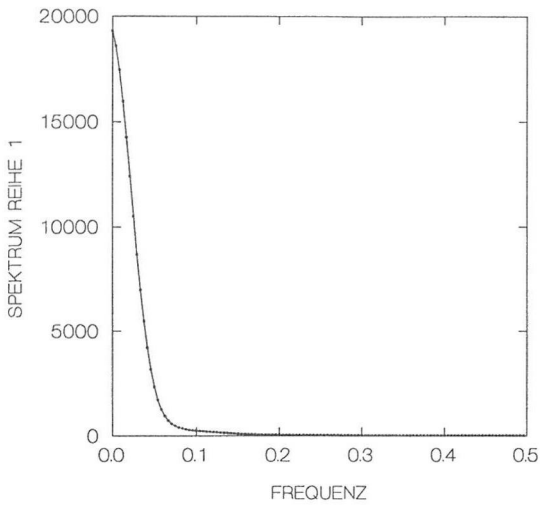


Figure 26

This spectrum offers no evidence whatsoever regarding the existence of a seasonal component. This result does not change even if one uses the logarithmic form of the spectrum (not presented here), which is sometimes to be preferred instead of the original spectrum, due to its higher resolution. Obviously, the entire spectral mass is concentrated in the low frequency domain, especially around the frequency f = 0. How may this be explained? The reason for this concentration is that the series exhibits a strong trend. A trend may, however, be interpreted as a fluctuation with an infinitely long period. Or expressed somewhat differently, its spectral mass corresponds to the frequency f = 0. This has important consequences for any spectral estimation procedure, as already indicated in the above section 3.3 under the rubric "leakage"-problem. Founded upon the trend, the observed concentration of spectral mass at f = 0 "trickles down" into the neighboring frequencies during the spectral estimation, thereby concealing (the actually existent) peak at f = 1/12.

This problem may, however, also be looked at from an alternative perspective. According to the theoretical fundamentals of spectral analysis, spectral analysis may only be applied to realisations of stationary processes. Such processes are characterized, among other things, by a constant expected value. Consequently, the realisations of such processes also fluctuate around some constant level. Series exhibiting a trend may therefore no longer be regarded as realisations of a stationary process. Thus, it is not permissible to apply spectral analysis to such series unless one previously removes the cause of non-stationarity of the time series. Specifically, this means that any trend must first be eliminated before spectral analysis may be applied. This presents us, however, with a new problem. How should trends be eliminated?

Unfortunately, as is obvious, there are no simple ad-hoc-solutions for this problem. One natural suggestion would be, for example, to model the trend by a polynomial of time and then estimate the unknown coefficients by the method of least squares. This method is unsuitable for a number of reasons. Given a specific time series, it is not a priori known whether or not a trend follows a polynomial. Furthermore, given that a series does follow a polynomial, the degree of the polynomial is as a rule unknown. This means that the degree must be more or less arbitrarily determined. In addition to these difficulties, least squares estimators present their own problems when applied to time series data, since various conditions are commonly not met, which makes least squares an inefficient procedure. It is common practice to attempt to eliminate a trend by taking first differences. As will be shown later (see Chapter II, Section 5.2), however, such a procedure may have *serious disadvantages*.

In general, the following postulate may be stated. A trend should be eliminated in such a manner, that all other components of the respective series are not affected by the transformation. With regard to the frequency domain, the above stated postulate implies that trend elimination should only affect those frequencies which are respective of the trend (that is, the frequencies f = 0 and the "direct" neighboring frequencies), all remaining frequencies should be unaffected. The above stated postulate is satisfied only by a "good" high pass filter. (Although taking first differences does indeed also represent a high pass filter, it is a very poor one as will be demonstrated below). This is not the proper place to present a detailed analysis of the construction of such filters. Therefore, we limit our discussion to demonstrating their effectiveness. (see further Chapter II, Sections 4 and 7).

Given an adequate high pass filter, one would expect that the filter-output of the series demonstrated in Figure 25 shows no observable difference when compared to the sinusoidal seasonal component. Therefore, Figure 27 shows both series together.

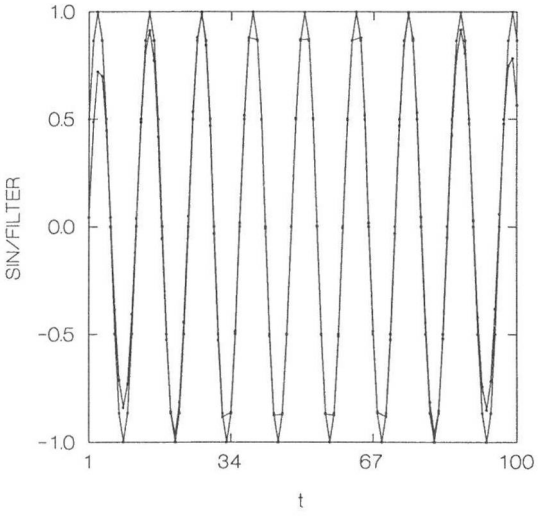


Figure 27

As simple inspection reveals, both series only show small dissimilarity differences at the beginning and end of the series.

Figure 28 now presents both the spectra of the filter-outputs and the sinus oscillation. Both spectra are practically identical.

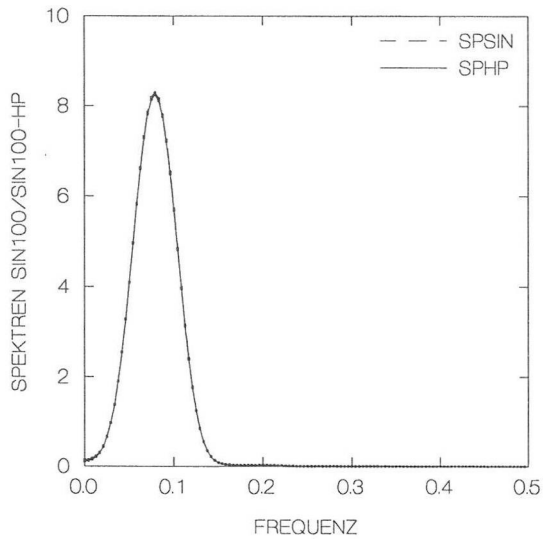


Figure 28

It will be demonstrated later on, that taking first differences does indeed eliminate any trend, however, at the same time may result in considerable distortions of the series.

Continuing our observation of a series with a trend, we now introduce additionally a noise-component as presented in the above Figures 15-17. The time series model is now given by

$$X_t = 2 + 0.9t + \sin(2\pi/12)t + \varepsilon_t \ , \ t = 1,2,...,100 \ .$$

Regarding Figures 29-31, notice that the seasonal component is barely observable anymore. Indeed, given the larger noise-variances it has disappeared entirely.

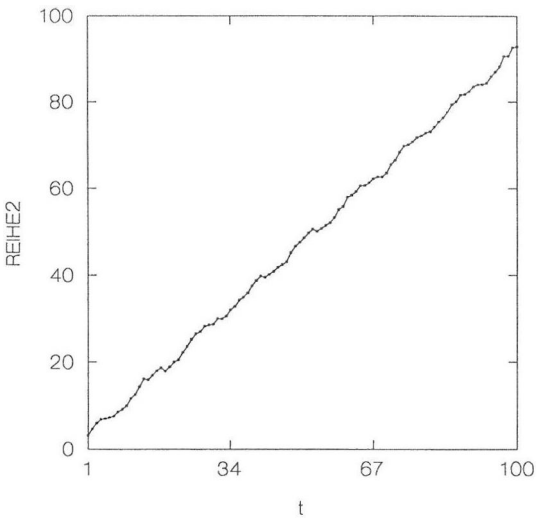


Figure 29

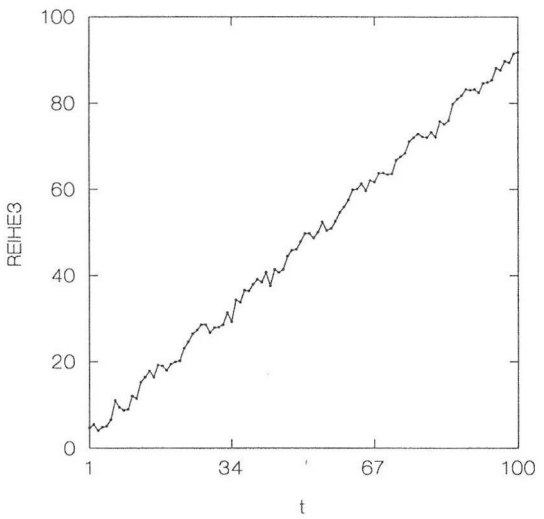


Figure 30

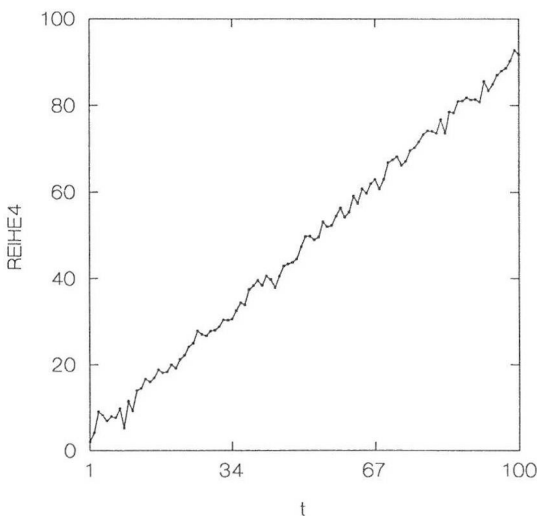


Figure 31

Of course, the spectra of these series (not presented here) also do not exhibit any seasonal component. In order to obtain informative spectra, a high pass filter must once again be applied. Figures 32-34 below present the corresponding filter-outputs together with the (seasonal + noise)-components. Observe here also that only minimal differences appear.

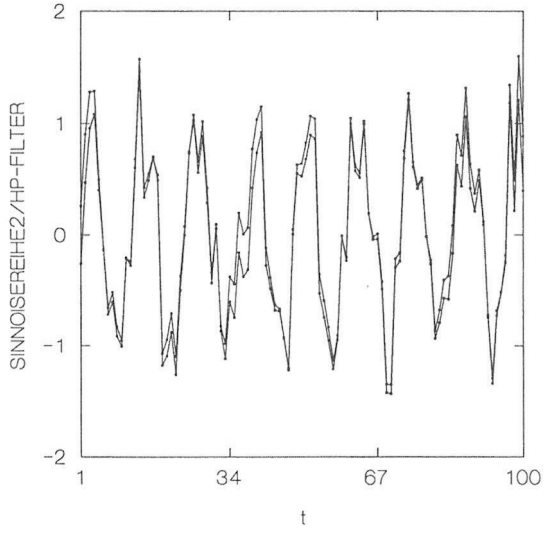


Figure 32

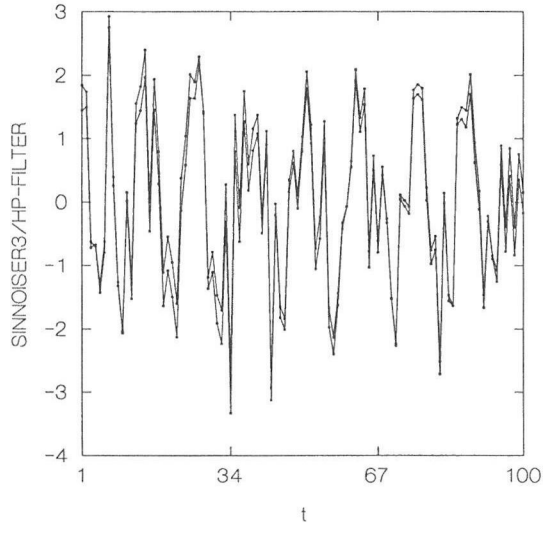


Figure 33

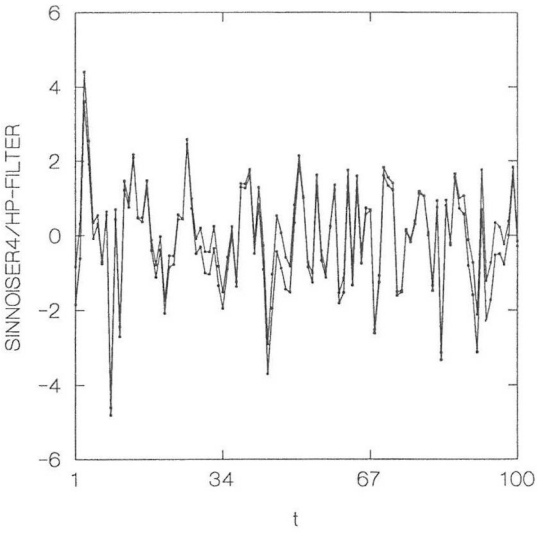


Figure 34

Thus as noted above, the corresponding filter-output spectra and the (seasonal + noise)-components are barely differentiated one from another. Therefore, it is no longer necessary to present them explicitly.

The previous results remain the same even if the series are based upon more complex components such as seasonal patterns containing harmonics, or assuming autocorrelated noise. As such, additional examples along these lines are superfluous.

In summary, therefore the above spectral analysis with simulated series demonstrates the following general conclusion.

Generally, determination of seasonality in economic time series that have a trend requires preliminary trend elimination under the application of an adequate high pass filter.

Adhering to this general conclusion, all of the analysized spectra of price series presented in Section 4.2 were based upon high pass filtered series.

4. Time Series Analysis of Price Quotations

4.1 Description of the Analysed Time Series

The following discussion applies data from the Swiss consumer price index for the basis months September 1966, September 1977 and December 1982 and data from the Swiss wholesale price index. The data were kindly made available by the Swiss Federal Statistical Office (National Economy and Prices Division, Prices and Consumption Section). Given the numerous amount of series available, the following discussion concentrates mainly on the Swiss consumer price index. This index reflects the development of relevant private consumer prices with respect to commo-

dities and services, thereby accounting for whether or not the costs of living have risen or decreased (without taking into account of possible influences due to changes in commodity quality and consumer behavior). The time period considered by the data used is December 1982 to the end of 1991. For specific inquiries, however, we also use the total index of consumer prices up to August 1992.

In the following presentation, the time series dealt with represent the main commodity groups of the Swiss consumer price index. For various main commodity groups of the food and manufacturing industries we also analyse some subcategories in more detail.

The list of time series examined contain the following:

Abbreviation:	*Commodity Group:*	*Symbol:*
PK	Total index	Totalind
PKA	Foreign	Ausland
PKB	Heating and Lighting	Heiz
PKD	Services	Dienstl
PKE	Education and Recreation	Bildung
PKG	Body and Health Care	Pflege
PKH	Household Furniture, Repairs	Haush
PKI	Domestic	Inland
PKK	Textiles	Kleid
PKM	Rent	Miete
PKN	Food	Nahrung
PKNC	Coffee	Kaffee
PKNE	Eggs	Eier
PKNF	Meat and Meat Products	Fleisch
PKNG	Vegetables	Gemüse
PKNH	Potatoes	Karto
PKNM	Milk	Milch
PKNO	Fruits	Frücht
PKNP	Fish	Fisch
PKNT	Grain Products	Getreid
PKT	Beverages and Tobbaco	Getraenk
PKV	Transport	Verkehr
PKW	Manufactures	Waren
PKWD	Durables	Warend
PKWN	Non-Durable	Warennd

4.2 *Empirical Results*

Applying spectral analysis, the available time series with price quotations are examined to reveal whether or not a seasonal component is present. Using a high pass filter [2] trends are first eliminated from the time series. Their spectra are then estimated. In order to detect any possible influence of the high pass filter procedure itself upon the series, an alternative follow up high pass filter was used [3]. The following results of spectra estimations were unaffected by the different filters. For purposes of clarity, only a few of the results from estimated spectra of selected time series are presented here. First, let us consider the time series for the Swiss consumer price index (TOTALIND), its respective estimated spectrum, the corresponding high pass filter adjusted time series and its designated estimated spectrum (cf. Figures 35-38). The spectrum of this time series exhibits a strong seasonal component with a cyclical length of 12 months.

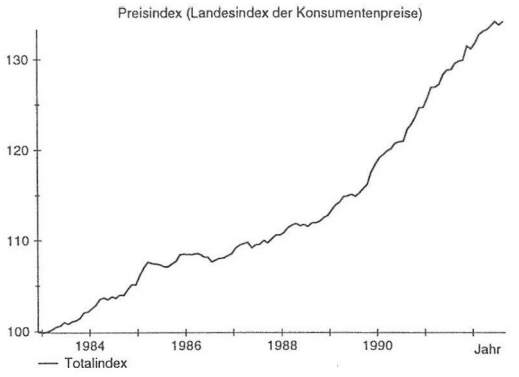


Figure 35

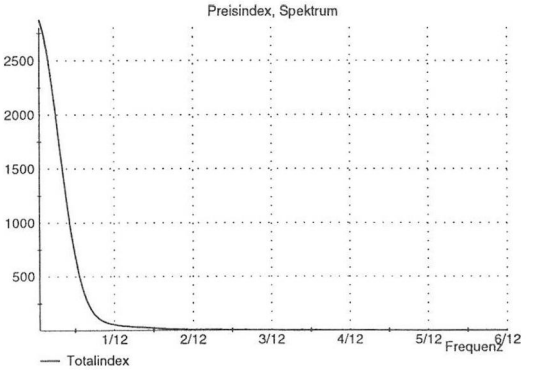


Figure 36

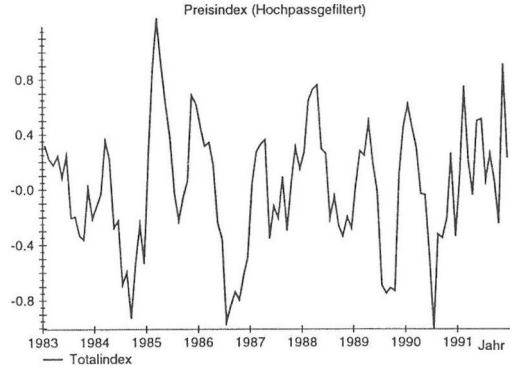


Figure 37

To some extent, diverging results for certain sub-indices may be obtained due in part to data collection and data-processing. The sub-index services (DIENSTL) displays a seasonal component with a cyclical length of 12 months as well as a strong additional component with a cyclical length of 3 months (cf. Figure 39). Inspection of the sub-index for textiles (KLEID) reveals basically only a single significant

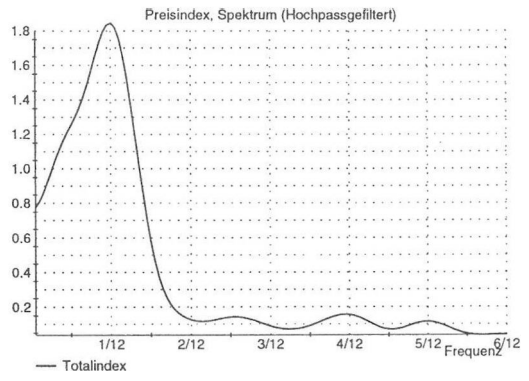


Figure 38

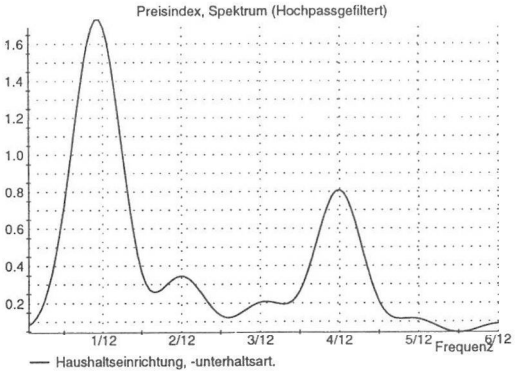


Figure 41

seasonal component with a cyclical length of 3 months (cf. Figure 40). With regard to the sub-index household furniture and repairs (HAUSH), the seasonal component exhibits a large degree of similarity with the services sub-index (cf. Figure 41). Due to data collection conditions, the sub-index for rents (MIETE) has a seasonal component with a 6 month cyclical length (cf. Figure 42).

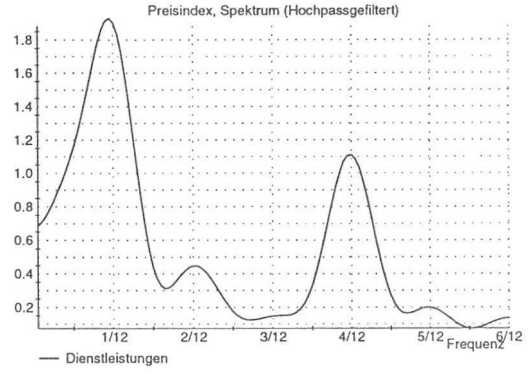


Figure 39

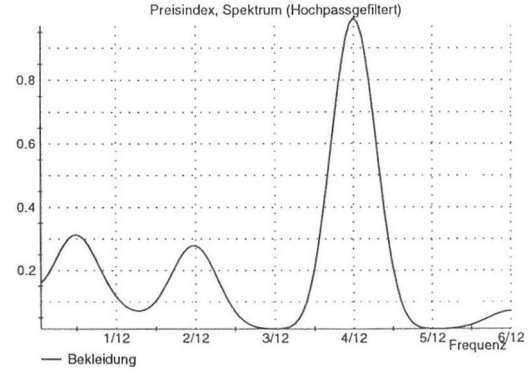


Figure 40

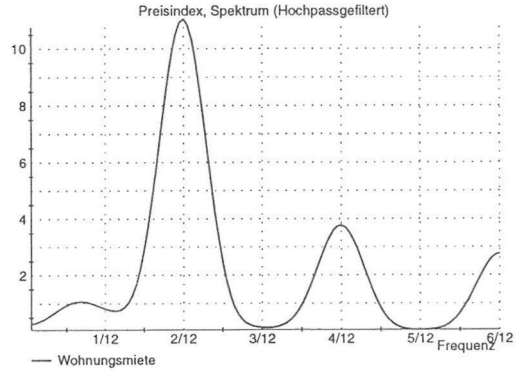


Figure 42

Figures 43-45 illustrate the estimated spectra of the adjusted high pass filtered time series for the sub-indices "education and recreation" (BILDUNG), "durable goods" (WAREND) and "Food, Beverages and Tobacco" (GETRAENK). These time series are characterized by the fact that they display cyclical components of varying lengths.

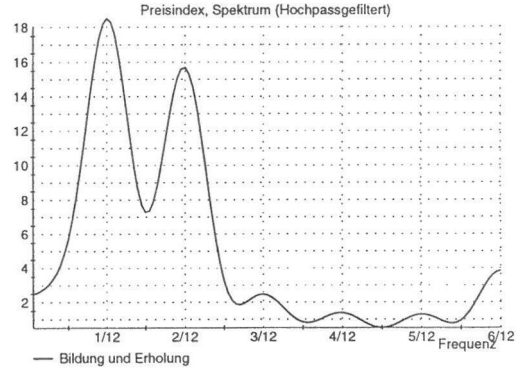


Figure 43

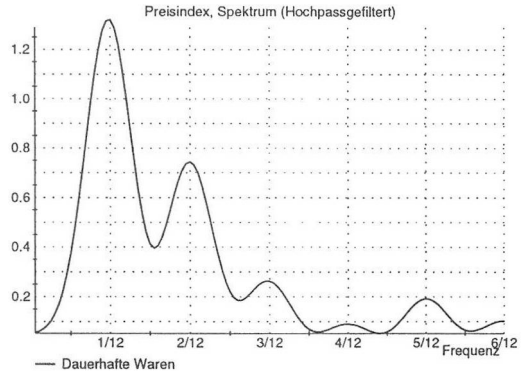


Figure 44

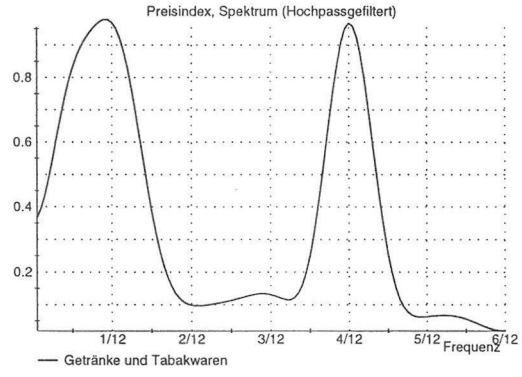


Figure 45

Literature

1) GRANGER C.W.J., HATANAKA M., Spectral Analysis of Economic Time Series, Princeton 1964
2) SCHMIDT R., Konstruktion von Digitalfiltern und ihre Verwendung bei der Analyse ökonomischer Zeitreihen, Bochum 1984
3) A so-called POTTER-Filter was alternatively applied. See OTNES R.K. et al., Applied Time Series Analysis, Vol. I, New York 1978, pp. 167-171

II. Design of Seasonal Adjustment Procedures

1. Objectives of Seasonal Adjustment

Earlier writings on the subject of time series of economic data were based upon the notion that these series consisted of various components. This idea goes back to practical experience collected in the areas of astronomy and meteorology in which decomposing time series into separate components was a common day practice [1]. In analysing economic time series, it is common to postulate the existence of a trend component, as well as cyclical, seasonal and so-called irregular fluctuations [2]. Despite wide usage of such decomposition methods, to date there exists no generally accepted concensus as to how respective components should be defined or in what manner they may be interrelated. Of course, this should not be very surprising considering the fact that it is not possible to verify definitively which model structure of an economic time series is to be accepted, since the separate components are principally unobservable. Furthermore, any attempt to associate various components as being defined by certain exogeneously observed variables is doomed to fail, due to the complexity and ambiguity of the relationships. For example, economic business cycle models offer a wide variety of competing definitions regarding cyclical fluctuations and their determinants. Any so-called "external" definition of components of economic time series therefore cannot be operationalized [3]. Given situations of practical economic time series analysis, it is thus necessary to choose a model structure that characterizes the particular components in a formal unequivocal manner with respect to the given data. This so-called "internal" definition of the components requires that one establishes economically plausible hypotheses regarding the supposed components. In doing so, the computation methods employed should be clearly established so as to correspond adequately well with the time series decomposition and its respective goal.

The aims and methodological criteria of time series analysis are of central importance to any relevant discussion of the topic [4]. Looking first of all at the aims and goals of time series analysis, one may first differentiate between retrospective and prospective objectives. With regard to the retrospective objective, one is dealing with given historical observation data which reflect some phenomenon to be examined in order to disclose the underlying data generating process. On the other hand, given a prospective objective, the time series examined is viewed primarily as a basis for diagnosing the state of the system for recent and current data and under certain circumstances for forecasting the future development of the system.

For quite some time, research interest focused mainly upon retrospective-orientated decomposition of series. Typical in this field are statements such as: business cycles are not characterized by strictly periodical fluctuations, but rather by fluctuations with changing periodocity or cyclical length and varying amplitudes. However, since the 1920's, such statistical research efforts have obtained a much more solid theoretical economic foundation. Considerable progress has been especially made when it comes to analysing questions of economic policy regarding short-run business cycle diagnostic procedures based upon current available time series data. Thereby, proper statistical analysis should be able to indicate the current economic situation, as well as any future developments of the cycle.

In order to establish such a diagnosis of the state of the system it is above all necessary to comprehend the significance of the trend and the cyclical fluctuations. An examination of both components in isolation is not only technically very difficult, it is also in principle not possible given methodological considerations. Consequently, both components are usually combined to construct a trend-cycle-component (commonly referred to as the "smooth" component). This aggregate component is also the one of main interest in diagnosing the business cycle. In fact today, the construction of smoothing methods that eliminate any seasonal and irregular fluctuations are often viewed as being the main purpose of economic time series analysis. Determining a seasonally adjusted time series is, however, usually only a procedural preliminary of time series analysis. Determining a seasonal movement component may, however, by all means also represent a proper exercise in itself. For example, when forecasting the production of seasonally dependent products, estimation of the seasonality of the series may help planning.

In any case, pursuing a prospective objective requires a carefully designed retrospective analysis of the underlying data generating process. In addition to having specific ideas with regard to the time series analysed, especially with regard to the stochastic nature of the individual components, one also needs criteria which permit judgement of the validity of the applied analysis procedures.

Although it may presumely be argued that for most applications a prospective objective prevails, that is, one is mainly interested in diagnosing the recent path of the smooth components especially with regard to their respective turning points in the form of local maxima and minima, there do exist deviating positions which should at least be mentioned here. In a strict sense, given a prospective objective, a seasonal adjustment is only an auxiliary mean of determining the smooth component of the most recent values at the boundary. Since as a rule, it is usually simpler to eliminate specific recurring seasonal fluctuations that exhibit a certain amount of regularity, rather than to attempt to estimate the respective value of the smooth component, seasonally adjusted values are sometimes referred to for purposes of diagnosis. This approach is, however, somewhat controversial. Opponents argue along the lines that the various components (especially the so-called irregular components) contain also important economic information that is lost if one restricts to simply observing and analysing the smooth component of the most recent values. If

this point of view is accepted, then it must also be concluded that applying seasonal adjustment techniques that model irregular components as white noise is inconsistent from a methodological perspective. White noise implies the notion that a process contains no further information whatsoever. Therefore, the interpretation of any analysis of a specific time series must also pay due attention to the time series modelling upon which it is founded.

2. A Brief Survey of Seasonal Ajustment Procedures

The decomposition of time series is founded upon the idea that an observed series consists of several components. The separate components are thereby associated with various causes. For semi-annual data, for example, monthly data, one commonly differentiates between four systematic and one so-called irregular component. The systematic components are then designated as the trend, cyclical, seasonal and calendar influence. The irregular component is as a rule interpreted as a residual component that may not be associated with anyone specific corresponding causative aspect. Sometimes it is also interpretated as a component representing outliers due to special influences to be found in the data. Often the exact time point of occurrence of such outliers may be determined, however, their quantitative relevance may at best be estimated. Any decomposition of economic time series data is thus confronted with the inherent problem that economic theory offers no direct definition that may be operationalized for formulating the above mentioned components, especially the systematical components. Therefore, time series analysis thus necessitates that conceptually complex causes must be described by theoretical models, which are then the basis for estimations of the non-observable components. Consequently, a decomposition of time series into separate components is always a statistical abstraction. As such, it depicts a more or less appropriate approximation of the verbally presented relationships of causation. In other words, the components based upon certain theoretical models of causation are simply affixed a specific semantically colored label. The question of whether or not the respective theoretical model decomposition of any given time series is adequate can not be solved by simple labelling of the components. From a methodological perspective, in order to obtain a solid judgement regarding any specific time series analysis procedure and its corresponding results it is imperative that one at least has detailed knowledge of the postulated model [5].

In determining which model structure is appropriate as a point of departure for a time series decomposition, the first problem that emerges is to decide the number and type of the associated components, as well as how they may be related (additive, multiplicative or mixed). The way in which they may be related is not deducible from purely economic considerations, rather the postulated relationship is always a constituent part of the choosen model approach. Historically, many procedures developed for time series analysis were often originally conceived without any explicit definition of a model structure. Naturally, this does not mean that the earlier procedures were not based upon some sense of a model with regard to the manner and relationships among the separate components. In running through the individual program steps of any analysis procedure, a model is always implicitly at hand, even if it is not possible after running through the various steps of the method to explicitly state the component modell [6]. The most well-known example for a procedure for which the basic model is implicitly determined only by a sequence of program steps is the CENSUS-X-11 procedure. The fact that this procedure is based upon an implicit definition makes a methodological judgement quite difficult and may under certain circumstances lead to inconsistencies with respect to the implementation of various procedural steps [7]. Given the case of the CENSUS-X-11 procedure, it is of particular interest here to note, that up till now only under the assumption of an additive relationship among components has it been possible to obtain a stochastic process capable of generating the data with corresponding optimal results [8]. Although preferably in practice one would expect a multiplicative relationship among components, as of yet no explicit compatible time series model has been successfully formulated that could serve as a basis upon which methodological judgement could be obtained.

For procedures with explicitly formulated models, one usually distinguishes between local and global models. Local models describe the properties of the postulated separate components only for partial intervals of the observation period [9]. Illustrative of such a local model is the BERLIN METHOD as applied by the German Federal Bureau of Statistics [10].

Without going into a detailed discussion of the historical development of time series analysis (we refer the interested reader here to the relevant literature [11]), the following discusses a few interesting steps in the development of time series analysis and sketches some of the main features of various renowned procedures. Naturally, the problem of seasonal adjustment will remain at the forefront of the discussion.

Many basic problems of seasonal adjustment may be demonstrated on hand of very simple time series models [12]. This is expedient for two reasons. First of all, procedures are commonly applied in practice that feature elements of these simple models. Secondly, in studying simple models first certain problems become transparent that also occur in the same or similar fashion when examining more complicated procedures.

Both of the following models discussed differ with regard to their pattern of seasonal figures. Whereas the first model exhibits a *constant* seasonal figure, the second model permits a certain degree of seasonal figure *variability* (to be

defined more specifically later on). Both approaches are based upon an *additive* component model.

For each time period t = 1,2,...,T (monthly, quarterly or half-yearly) respective observation values X_t of some metric feature are given (e.g. product sales turnover, number of unemployed etc.). Hereby, the notion that the observed time series values are constructed additively from different components is fundamental. This component model may now be formulated as

$$X_t = T_t + Z_t + S_t + U_t \quad , \quad t = 1,2,...,T \quad .$$

T_t is the trend component, whose path is considered to be determined by long run effects. It is often assumed, that the trend increases or decreases monotonically. Z_t is the cyclical component, whose path is assumed to be contingent upon the business cycle.

Often T_t and Z_t are not analyzed separately. The sum of both these components is then commonly designated as the *smooth component* G_t. S_t is the seasonal component, whose path is determined by seasonal and institutional factors.

Saisonal adjustment now implies that S_t can be identified and eliminated. It is obvious, that seasonally adjusted series are of great importance for economics.

U_t is the irregular component. U_t is regarded to be determined by factors other than those mentioned above. Further, it is assumed that the values for U_t are relatively (i.e. in comparison to the values of G_t and S_t) small and fluctuate irregularly around the value zero. The U_t's represent residual fluctuations that possess the character of *stochastic fluctuations.* Therefore, they are usually interpreted in time series models with corresponding seasonal adjustment procedures as white noise.

If one assumes that the seasonal pattern is constant, i.e. it holds that

$$S_t = S_{t+12} \ , \ t = 1,2,...$$

implying strict periodocity of seasonality, then by adding two further suppositions a very simple seasonal adjustment procedure may be constructed [13]. The additional assumptions are:

a) the smooth component G_t may be sufficiently approximated by a linear function over a period of 13 months,

b) the sum or weighted average of the values of the irregular component may be replaced with sufficient exactness by the value zero. It is useful to standardize the seasonal component to zero, that is to standardize the 12-term sequence $(S_1, S_2,...,S_{12})$ to $S_1 + S_2 + ... + S_{12} = 0$. The generality of this approach is not limited in any respect by this standardization since a constant can simply be subtracted from the S_t values and then added to the smooth component.

Given the above supposition a), the smooth component may now be estimated with the help of a moving symmetrical 12-month average,

$$\hat{G}_t := \frac{1}{12} \left[\frac{1}{2} X_{t-6} + \frac{1}{2} X_{t+6} + \sum_{i=t-5}^{t+5} X_i \right] ,$$

since the seasonal and residual components under supposition b) are assumed to be approximately zero.

$\hat{G}_t$ obviously represents a symmetrical filter. It does, however, have the disadvantage that the first resp. the last six values at the beginning resp. at the end of the series are disregarded, thus the estimation of the smooth component does not reflect the most recent observations at the *boundary.* Since the practical interest often is focused on the most recent data values, this disadvantage is a rather serious one. This filter may of course be conceived as being asymmetrical, thereby formally including the most recent data observations in the estimation. However, this apparent solution also has the disadvantage that such a filter implies a phase shift, which depending upon the filter construction used may be more or less important with regard to the results obtained. This vividly demonstrates the so-called *boundary-problem* of seasonal adjustment procedures that also emerges by more complicated procedures, creating a number of practical and theoretical difficulties.

Since $\hat{G}_t$ is an estimator for G_t, it holds that

$$X_t - \hat{G}_t \approx S_t + U_t \quad .$$

The averages of the S_t- and U_t-values may now be derived for the respective monthly observations. Let X_{ij} denote the series value of the j-th month in the i-th year and $S_{ij} = Sj$ due to the postulated constant seasonality. Thus,

$$\tilde{S}_j := \frac{1}{n_j} \sum_i (X_{ij} - G_{ij}) \approx S_j + \frac{1}{n_j} \sum_i U_{ij}, \ j = 1,2,...,12$$

n_j denotes the number of years, for which $\tilde{S}_j$ may be calculated. The standardization of the twelve values $\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_{12}$ is achieved by subtracting the arithmetic mean

$$\hat{S}_j := \tilde{S}_j - \frac{1}{12} \sum_{j=1}^{12} \tilde{S}_j \quad , \ j = 1,2,...,12 \quad .$$

Finally, the seasonal adjusted series is obtained by taking the differences $X_{ij} - \hat{S}_j$.

This simple approach was, for example, implemented in the 1930s by the U.S. Federal Reserve Board and the London Economic Service [14].

The simplest case of a variable seasonality is given when the seasonal component for the month j is assumed to be proportional to the smooth component [15],

$$S_{ij} = \lambda_j G_{ij} \quad .$$

In the previously discussed procedure, the absolute values of seasonal fluctuations were constant for months of the same name. In the present procedure, the relative magnitudes (with respect to the smooth component) are constant. Given this context, the additive time series model can be formulated as

$$X_{ij} = G_{ij} + \lambda_j G_{ij} + U_{ij}$$

$$= I_j G_{ij} + U_{ij} \quad , \quad I_j = 1 + \lambda_j \quad .$$

I_j represents the seasonal factor for the j-th month. Under the same conditions as above, one obtains

$$\frac{X_{ij}}{G_{ij}} = I_j + \frac{U_{ij}}{G_{ij}}$$

resp.

$$\frac{X_{ij}}{\hat{G}_{ij}} \approx I_j + \frac{U_{ij}}{\hat{G}_{ij}} \quad .$$

Now averaging these ratios over all respective available years for the same months, the following estimator for the seasonal factors are obtained:

$$\tilde{I}_j = \frac{1}{n_j} \sum_i \frac{X_{ij}}{\hat{G}_{ij}} \quad , \quad j = 1,2,...,12 \quad .$$

Hereby, the $\tilde{I}_j$'s are now standardized to the sum 12. The estimated seasonal factors are then,

$$\hat{I}_j = \tilde{I}_j \frac{12}{\sum_{j=1}^{12} \tilde{I}_j} \quad , \quad j = 1,2,...,12 \quad .$$

Finally, the seasonal adjusted series is obtained by the ratios $X_{ij} / \hat{I}_j$. The boundary problems are the same here as presented above for the model with constant seasonality.

Given real time series, a constant seasonality or even a variable seasonality of the kind as discussed above, will seldomly occur - if at all. However, both presented procedures do make evident that even given the simplest forms of seasonalities, various assumptions are necessary without which the identification and elimination of the seasonal component would be impossible.

It is also obvious that without special postulates a decomposition will not be possible. For each t only one series value is observed. Thus, for example, if the series is decomposed into four components, then one obtains 4T unknowns for T equations. The number of unknowns is always greater than the number of equations.

More complicated adjustment procedures that assume a more or less irregular seasonality, are founded upon other resp. more critical assumptions regarding the patterns of the individual components. For example, sometimes it is postulated that the smooth component can be modelled by a polynomial of time of at least the third degree (in order to model turning points) and whose corresponding coefficients may be estimated by a regression approach. But new problems consequently arise. Updating, that is taking consideration of new observations, may lead to large changes in the estimated regression coefficients, so that the previously adjusted values - especially with regard to the most recent observations - may also change significantly. This might have the consequence that the last "diagnostics" must be revised. In practical situations, this may be especially undesirable. This leads once again to the above mentioned problem of boundary stability, which often is a serious problem in many practically applied procedures.

Now let us turn our attention in the following to the CENSUS-X-11 procedure of the Bureau of the Census. First of all, note that "the" CENSUS-X-11 procedure as such does not exist. Rather, the U.S. Bureau of the Census developed since the 1950's a series of procedures which themselves are based upon methods developed during the 1920's and 1930's by the Federal Reserve Board and the National Bureau of Economic Research. Basically, these early procedures applied ratio-to-moving-average-methods, i.e. a multiplicative time series model was postulated. Thereby, the smooth component is estimated by a 12-month moving average and the ratios

$$G_t \cdot S_t \cdot U_t / \hat{G}_t \approx S_t \cdot U_t$$

smoothed for months of the same name. In doing so, it is assumed that the irregular component is "*ironed out*", permitting an estimation of S_t. Dividing X_t by the estimated seasonal component $\hat{S}_t$ yields the seasonal adjusted series. Choosing an additive approach rather than a multiplicative one, one could proceed analogously by simply replacing the ratios by differences. As should be apparent, the essence of this procedure is basically quite similar to the simple procedures we discussed above.

The above seasonal adjustment procedures were carried out manually by the Bureau of Census until 1954. During this year for the first time, a special computer program called CENSUS I was introduced. It was modified during 1955 and became known as CENSUS II. To date, many variants of these original programmes have been developed right up to the common day CENSUS-X-11 procedure still presently in use [16]. The symbol X indicates the experimental nature of the procedure, alluding to the fact that the procedural variants are not founded upon theoretical time series models

but rather are derived experimentally. At the same time, this is also one of the common criticism raised against the X-11 procedure. All the same, it remains to say that the CENSUS-X-11 procedure is without doubt the most widely used seasonal adjustment procedure worldwide.

Unfortunately, since the CENSUS-X-11 procedure is not based upon an explicitly written down time series model it is not possible to give a complete statement of the procedure. One can merely define the procedure based upon a sequence of procedural steps. Basically, two steps may be differentiated that vary with regard to their construction as well as their objectives. In step 1, a so-called *preliminary* component estimation is carried out in order to identify and eliminate *extreme values*. The *final* component estimation is then conducted in step 2 leading to the *actual* seasonal adjustment. In addition to the elimination of extreme values (i.e. the identification and correction of outliers), the procedure also employs a so-called *trading day* adjustment (i.e. the correction of any calendar irregularities) as well as an elimination of *singular* effects (e.g. caused by the occurrence of strikes). We omit presenting these adjustments in detail here. Thus, neglecting the above mentioned special adjustments, the main steps of CENSUS-X-11 procedure may be listed as [17]:

a) compute the differences between the original series and a centered 12-term moving average as a first estimate of the seasonal and irregular components. Six values at each end of the series are lost

b) apply a weighted 5-term moving average to each month separately, to obtain an estimate of the seasonal component. For the next to last 12 values use an asymmetric 4-term moving average

c) adjust these seasonal components to sum to zero (approximately) over any 12-month period by subtracting a centered 12-term moving average from them. To obtain the six missing values at the end of this average, repeat the last available moving average value six times

d) subtract the adjusted seasonal component from the original series, to give a preliminary seasonally adjusted series. The seasonal component for the last six values is missing as a result of step a): for these 6 months use the estimated seasonal component for the corresponding month of the preceeding year

e) apply a 9-, 13- or 23-term Henderson moving average to the seasonally adjusted values, and subtract the resulting trend-cycle series from the original series to give a second estimate of the seasonal and irregular components. At the end of the series use equivalent asymmetric moving averages

f) apply a weighted 7-term moving average to each month separately, to obtain a second estimate of the seasonal component. At the end of the series use equivalent asymmetric moving averages

g) repeat step c)

h) subtract these final estimates of the seasonal component from the original series, giving the seasonally adjusted series.

The above is based upon the additive version of the procedure. The multiplicative version commonly used in practice requires one to make certain relatively small modifications pertaining to the above list (instead of subtraction division, adjusting to twelve instead to zero, etc.).

In addition to the actual seasonal adjustment, the CENSUS-X-11 procedure also offers a number of tests that allow the user to further confirm the goodness-of-fit and evaluate the applied adjustments.

Even though the CENSUS-X-11 procedure is widely used and commonly characterised as *successful*, it has also aroused strong criticism. The main points are:

- a lack of analytical transparency since it is not founded upon an explicitly formulated times series model

- it uses symmetric, time-invariant filters in the *series center* and asymmetric, time-dependent filters for the boundary, especially for the current data. Given new data, this makes revisions necessary, thus reducing the reliability of such estimations in comparison to those in the center. Furthermore, under certain circumstances, such filters imply phase-shifts of unknown magnitude. (With regard to this aspect, see the discussion in chapter III.)

- the various *special adjustment procedures* (outlier-, trading day- etc. adjustment) are based upon subjective notions, which make the procedure even less transparent.

- looking at the procedure as a whole, it possesses too many *screws* that the user may *turn* at will. In addition to this, one is confronted also with the fact that various official users of the procedure prefer their own *home-made versions* (e.g. the German Bundesbank). This has the effect that different versions of the procedure are often being implemented under the same name "CENSUS-X-11". Consequently, series adjusted by institutionally modified CENSUS-X-11 procedures series are commonly not comparable.

On the one hand, the criticism of the CENSUS-X-11 procedure initiated research attempting to create a theoretical *foundation* of the procedure. However, at the same time, critique has also led to further development of the procedure.

For example, the classical additive component model may be expressed in the following manner

$$X_t = S_t + N_t \ ,$$

with N_t summing up the non-seasonal components. In this model, *seasonal adjustment* is equivalent to estimating S_t, i.e. the solution of a signal-extraction problem. The error of estimation thereby is

$$\delta_t = S_t - \hat{S}_t = \hat{N}_t - N_t \ .$$

An adjustment procedure may be regarded to be *optimal*, given a respective loss function for which the expected loss is minimal. Given a squared loss function and an estimation function linear in observed values X_t, it holds that

$$\hat{S}_t = \upsilon(B)X_t = \sum_{j=-\infty}^{\infty} \upsilon_j X_{t-j}$$

minimizes the mean squared error, given that X_t represents an infinitely long stationary stochastic process. The operator $\upsilon(B)$ is obtained by an inverse Fourier-transform of the ratio of the spectra of S_t and X_t [18].

Since $\hat{S}_t$ represents a moving average, the question arises whether there are connexions here with the moving averages of the CENSUS-X-11 procedure. Unfortunately, since economic time series seldomly may be interpreted as stationary processes, the above result may not be directly put to practical use in solving this question. However, it is often possible to model such series with the aid of the ARIMA-approach, implying that the series is regarded to be a realisation of a (homogeneous) non-stationary process that has been transformed into a stationary process by taking differences.

CLEVELAND and CLEVELAND-TIAO have shown that the above optimization problem may also be solved for ARIMA-processes, and that the solution has the same form as above [19]. Given additive time series models, this may therefore be regarded as an indication that the utilization of moving averages for seasonal adjustment may be appropriate. The extensions of the CENSUS-X-11 procedure applying ARIMA-models are along these lines of thought.

Characteristic of CENSUS-X-11-ARIMA is the combination of the CENSUS-X-11 procedure with forecasts based upon ARIMA-models. The basic idea is simple. In order to use symmetric filters at the boundary, one could simply estimate the respective future series values, for example, with the help of ARIMA-models. CENSUS-X-11-ARIMA consists basically of the following steps [20]:

a) the series to be adjusted is accomodated for an ARIMA-model

b) with the help of this model the series values are forecasted for the next 12-24 months and the original series is respectively elongated

c) this elongated series is then seasonally adjusted with the CENSUS-X-11 procedure.

Today, CENSUS-X-11-ARIMA is e.g. the official adjustment procedure used by Statisitics Canada.

Basically, practical implementation of a procedure requires that it is largely standardized. Since for a large scale application an automatic version is necessary, the problem of identification and estimation of potential ARIMA-models has to be solved in an economically efficient manner.

In comparison with CENSUS-X-11, it is argued in favour of CENSUS-X-11-ARIMA that considerably less revisions at the boundary are necessary. This has been documented by detailed studies [21].

As this report was written, the long announced ARIMA-extension of the X-11 procedure by the US CENSUS Bureau, the so-called X-12-ARIMA version, was still not available for detailed analysis [22]. Apparently, the programming of a PC-compatible version is causing some difficulties [23].

The SABL-procedure is quite similar to the CENSUS-X-11-procedure [24]. Both are iteratively structured and both use a variety of smoothing procedures which have not been formulated explicitly within a well-defined conceptional time series model, but correspond to general smoothing criteria. In SABL an additive as well as a multiplicative version is available. A detailed presentation of the SABL-procedure here is beyond the scope of this report.

However, the main difference as compared with the CENSUS-X-11 procedure is that the SABL procedure uses resistant smoothing methods. Such methods have the property of being resistant to outliers, that is, to observations which differ largely in magnitude compared to the majority of observations. As such, for example, a moving average is not a resistant smoothing procedure, whereas a moving median is a resistant one. An additional feature of SABL is its heavy usage of graphical methods for evaluating the "quality" of adjustments.

The BERLIN METHOD [25], currently with four existing versions, is the official adjustment procedure used by the German Federal Bureau of Statistics (Statistisches Bundesamt).

Here, the basis is also the classical model in which it is assumed that the components may be modelled as stochastic processes, for whose realisations it holds that

$$G_t = \sum_{j=0}^{p} a_j t^j \ ,$$

$$S_t = \sum_{k=1}^{6} (b_k \cos \lambda_k t + c_k \sin \lambda_k t) ,$$

i.e for the smooth component G_t a polynomial of time of degree p and for the seasonal component a trigonometric polynomial with seasonal frequencies $\lambda_k = 2\pi k/12$, k = 1, ...,6 is postulated. It is assumed that the residual component ε_t may be characterized by a weak stationary process with $E(\varepsilon_t) = 0$. Therefore, one can write

$$X_t = \sum_{j=0}^{p} a_j t^j + \sum_{k=1}^{6} (b_k \cos \lambda_k t + c_k \sin \lambda_k t) + \varepsilon_t.$$

One feature of the above procedure, for example, is that it does not postulate the validity of the model for all time points, but rather only for a subset of time points covering the interval [1,T] in a sliding manner. In the above definitions of G_t and S_t it is therefore generally permissible to require various lengths N_1 resp. N_2 for the sub-intervals.

Given the situation where a time point may lie in more than one supporting span, a selection criterion is needed. In principle, as such *variance minimization* is choosen. Unfortunately, this criterion does not always lead to unique solutions and may also not be acceptable for various other reasons.

The smooth and seasonal component should be estimated simultaneously. In practice, this is not the case since the supporting spans N_1 and N_2 for economic series must be assumed to differ strongly ($N_1 < N_2$).

Consequently, in practice the smooth component is estimated first (supporting span N_1) and then subtracted from the original series. Then the seasonal component is estimated from the original series which has been adjusted for the smooth component (supporting span N_2). By subtraction of the estimated seasonal component from the original series one obtains the seasonal adjusted series.

As usual with regression approaches - given the procedure should be of practical use - specification problems must be solved. Basically, this implies the determination of the degree p, the length of the supporting spans N_1 and N_2, as well as the supporting points τ_1 and τ_2 within the supporting spans. As a measure of the goodness of fit for a chosen specification one examines above all the properties of the resulting filters in the frequency domain, especially their amplitude functions (the above mentioned goodness-of-fit criterion *variance minimization* is hereby only of secondary importance). The amplitude functions are judged based upon a *reference amplitude function* that does not exhibit any undesired attenuations or amplifications and were therefore considered to be *ideal* by the developers of the BERLIN METHOD. This one takes on exactly the value one in the low frequency interval [0, 1/12) and exactly the value zero at seasonal frequencies 2k/12, k = 1,...,6, exhibiting a pattern of five *notches*. Between the seasonal frequencies it takes on the value one (see further Section 4 in this chapter).

In version I of the BERLIN METHOD more than some 200 filters were analysed with regard to their amplitude functions. It was always the case that undesirable attenuations resp. amplifications were observed. The following specification was finally considered *favorable* for the *center*: p = 3, $N_1 = 23$, $\tau_1 = 13$, $N_2 = 45$, $\tau_2 = 24$. Asymmetric filters were applied at the boundary.

The subsequent development of the original version was carried out in Germany by the Federal Bureau of Statisitics (Statistisches Bundesamt), who nowadays uses its version 4. All versions may be regarded as attempts to better approximate the ideal amplitude function. For this purpose, Version 2 introduced in addition to the time-polynomial a trigonometric polynomial with a period of 36 months in the hope of improving modelling the business cycle. Furthermore, support spans and support points applied in estimating smooth and seasonal component were modified in this version. Version 3 is of relevant interest because the concept of a standardized adjustment procedure was abandoned [26]. Since the applied filters were not free from distortions, one had to make allowance for the fact that such distortions could affect different series very differently. In addition, the stability at the boundary of the previous versions was considered to be insufficient. Therefore, the selection of a filter in this version has been based upon the series to be adjusted (*series specific adjustment*). A filter is considered to be *optimal* if it produces an adjusted series whose (estimated) spectrum shows the smallest deviation (in the least-squares sense) from the (estimated) spectrum of the original series for all frequencies except the seasonal ones. This is equivalent to saying that a filter should be selected whose amplitude function is as close as possible - in the above sense - to the ideal amplitude. In Version 4, however, the German Federal Bureau of Statistics returned to a standardized procedure. In principal, the same methods were used as in the previous versions, but with two innovations. In estimating the smooth component, a least squares approach is taken that uses a symmetrical weighting function for the center and an asymmetrical triangular function for the boundary. The similarity to the SABL-procedure should be apparent. The second innovation involves at the boundary a successive substitution of the third degree polynomial by a combination of two polynomials of the third resp. first order, whereby the weight of the linear approach increases towards the end of the series. By so doing, the boundary stability of the procedure is improved.

Among recent developments procedures developed by AKAIKE and KITAGAWA/GERSCH, PAULY and SCHLICHT are to be mentioned. The fundamental idea originally was conceived by WHITTAKER as a solution proposal of the following problem [27].

Given the time series model:

$$X_t = f_t + u_t \quad , \; t = 1,...,T \; ,$$

whereby f_t is an unknown smooth function of time and u_t represents identical and independent random variables with an expected value of zero and constant variance σ^2. The function f_t describes the systematic component of the time series and is to be estimated under the following two constraints:

- the estimations of f_t should deviate as little as possible from the values of the given series
- the function f_t should be as smooth as possible.

Since both of the above restrictions are contradictory, they may not be simultaneously realized. WHITTAKER [28] proposed a compromising solution which considers minimizing the following objective function:

$$\sum_t [X_t - f_t]^2 + \lambda^2 \sum_t [\Delta_k - f_t]^2 ,$$

with k denoting the order of the difference operator Δ and λ^2 representing a calibration parameter. The first term of the sum expresses the differences between the series values and the values of the systematic component f_t. The second term is a measure of curvature f_t. The parameter λ^2 that has to be choosen characterizes the necessary compromise between goodness-of-fit and smoothness.

AKAIKE and SCHLICHT have extended this approach to time series with seasonal components. Thereby, the systematic component is an additive combination of the smooth component G_t and the seasonal component S_t. The *Seasonal Adjustment Principle* proposed by SCHLICHT consists of the following three postulates:

a) the sum of smooth and seasonal component should approximate the given series as well as possible, i.e. the irregular component u = x - g - s should be as small as possible [x, g, s, u are T × 1-vectors, e.g. $x' = (X_1, X_2,...,X_T)$]

   Given that $f_3 : \mathbb{R}^T \rightarrow \mathbb{R}$ is a measure for the size of u (e.g. $f_3(u) = u'u$), then taking the minimum of $f_3(x - g - s)$ represents an operationalization of postulate a).

b) g should preferably be a very smooth curve. Taking $f_1 : \mathbb{R}^T \rightarrow \mathbb{R}$ to be a (arbitrary) measure of curvature. Postulat b) requires the minimization of $f_1(g)$

c) the seasonal component should exhibit a development pattern as stable as possible and in addition average to zero over the seasonal cycle.

   If $f_2 : \mathbb{R}^T \rightarrow \mathbb{R}$ represents a measure of instability of the seasonal pattern and of deviation from zero, then postulate c) requires a minimization of $f_2(s)$.

If the functions f_1, f_2, f_3 are given, the smooth and seasonal component may be determined as the solution of the following minimization problem:

$$V(g, s \mid x) := f_1(g) + f_2(s) + f_3(u) = \min_{g,s}.$$

SCHLICHT [27] suggests the following functions:

$$f_1(g) := \alpha \sum_{t=3}^{T} [(G_t - G_{t-1}) - (G_{t-1} - G_{t-2})]^2 , \quad \alpha>0$$

$$f_2(s) := \beta \sum_{t=s+1}^{T} (S_t - S_{t-s})^2 + \gamma \sum_{t=s}^{T} (\sum_{\tau=0}^{s-1} S_{t-\tau})^2 ,$$

$$\beta \geq 0 \, , \gamma > 0 \qquad (s = 12 \text{ for monthly data}).$$

$$f_3(x\text{-}g\text{-}s) := \sum_{t=s+1}^{T} (X_t - G_t - S_t)^2 .$$

The first term in the summation of $f_2(s)$ measures the stability of the seasonal component and the second measures the sum of the yearly deviations of this component from zero.

The above minimization problem has a unequivocal solution for the choosen specification. This procedure also provides degrees of freedom in so far as α, β and γ have to be specified numerically. The larger α and β are, the smoother is G_t resp. the more stable is S_t and naturally the larger is the irregular component. Instead of simply determining these parameters ad-hoc, they may be specified so that their ex-post forecasting error for s and g are minimal. The latter are defined by comparing the results (for a given set of coefficients) till period t < T with those of the entire available series and considering the corresponding differences. The respective set of coefficients that minimizes this forecast error is then regarded as "optimal".

The theoretical considerations regarding the CENSUS-X-11 procedure and the various versions of the BERLIN METHOD, which analyse the amplitude function directly as the "reference value", offer impulses for a somewhat different construction of seasonal adjustment procedures. The term "direct" implies that the frequency domain filter is constructed without preliminary specification of the components and estimation methods in the time domain. Such a pure filter-theoretical concept of a seasonal adjustment procedure is no longer founded upon the classical component model. Instead of the customary traditional four components conceived, now an infinite number of a priori components corresponding to the continuum of frequencies [0, 1/2] is permissible. For purposes of interpretation and formulation of objectives it is often, however, expediant to consider frequency-intervals and to apply common terminology to them. For example, it is obvious to identify the interval [0, 1/12) with the smooth component (in the sense of the

traditional component model). This approach will be discussed in more detail in Chapter IV.

3. Criteria for Seasonal Adjustment Procedures

There exists quite a variety of goodness-of-fit and evaluation criteria for seasonal adjustment procedures. As it were, many evaluation criteria and concepts were developed parallel to methodological improvements. The most commonly used criteria differ not only with respect to their degree of complexity, but also with regard to their theoretical foundations. Consequently, simple direct comparisons of procedures are not always possible.

For example, one simple pragmatic criterion is to demand the invariance of the annual sum. This is commonly referred to as the criterion of sum constancy. This criterion requires that the sum of the seasonal adjusted values over the time period of a year should equal the sum of the unadjusted values. Although apparently a sensible and clearly intelligent criterion in practice, it generally is not suited to be used for comparisons of procedures. Some procedures are founded upon a normalization of the seasonal pattern so that the above procedure is fulfilled automatically. Furthermore, it is always possible to modify a procedure so that this criterion holds.

The question thus emerges, "What are potential criteria candidates for evaluating and comparing different procedures?" The known criteria may be, first of all, classified according to whether or not they are defined in the time - or in the frequency domain. Examples for criteria stated in the time domain are the orthogonality property and the idempotency property [29]. The orthogonality property ensures that a seasonally adjusted series contains no further information that may be used to determine the seasonal component, i.e. the removed part of the series is empirically uncorrelated with what is left after the adjustment. Violation of this criterion would be circumstantial evidence that a seasonal adjusted time series still contains seasonal movements. The idempotency property demands that an adjustment procedure leaves a series unchanged that was previously adjusted by the same method.

Practical experience has demonstrated, however, that usage of the above mentioned criteria and similar criteria do not permit an adequate discrimination between various common seasonal adjustment procedures. It is often the case that different procedures fulfill such criteria more or less to the same extent. The above statements also apply to a number of other criteria that deal with the resulting residual component as estimator of the irregular component. This category of evaluation-criteria includes, for example, run-tests or other tests of randomness such as periodogram tests etc. In addition to this, it must be noted that all criteria that are based upon the residual component are applicable only for procedures that emanate from a traditional component model.

The above mentioned criteria implicitly assume a retrospective objective. Given a prospective objective, i.e. attempting to evaluate the accuracy of a procedure to correctly diagnose the fundamental cyclical tendency for the boundary, special criteria have also been developed [30]. The basis for such criteria is a diagnosis of the system state at the boundary of a time series given a specific procedure. These findings are then compared to those of a retrospective analysis obtained later by applying the same procedure and evaluating with the aid of suitable criteria.

In addition to criteria conceived in the time domain, evaluation criteria for seasonal adjustment procedures have also been developed for the frequency domain. For example, it may be required that the spectrum of an adjusted series does not exhibit "peaks" or "troughs" in the seasonal frequencies and that it corresponds to the spectrum of the observed time series in the low frequency domain. "Peaks" in the seasonal frequencies are hereby interpreted as an insufficient adjustment, "troughs" as an overadjustment. A modified spectrum in the low frequency domain is interpreted as a distortion of the trend-cycle component caused by the applied seasonal adjustment. It is furthermore required that the coherency function with respect to the observed time series and the adjusted series takes on a value as close as possible to one within the non-seasonal frequency bands and a zero value in the seasonal frequencies. These criteria, as well as other similar spectral criteria, are also subject to criticism. Above all, it also proves to be the case for these criteria as well that they are only suitable for comparison and evaluating adjustment procedures to a certain degree [31].

In summary, each of the above discussed criteria only can grasp partial elements of any procedure. This leads to the suggestion of searching for a catalog of criterion that would permit effective evaluations of the various procedures. The choice and weighting of the individual criteria is, however, controversial among researchers. Furthermore, an objective hierarchial ranking of the individual criteria is not possible. Consequently, it comes as no surprise - perhaps to the disappointment of many practical users - that based upon such criteria lists there still exists no "best" procedure.

A first step in trying to solve this user problem is not to observe isolated aspects of a specific procedure, but rather to attempt to analyse the properties of the procedure as a whole. However, since the methodological conceptions of the various procedures are often quite distinct from one another, it is first necessary to look for some common denominator for all procedures. Inspite of all methodological differences of individual adjustment procedures, each one may be expressed as a filter that transforms an observed time series into an adjusted series. This suggests an evaluation in the frequency domain. In so doing, to begin with the filter properties such as amplitude - and phase functions are only explicitly known for those procedures that were also constructed with the aid of filter theory. For all other procedures the total filter effect must first be

ascertained. Analytically, this may be quite difficult in certain cases and only approximative, especially if the procedure consists of many procedural steps whose sequence is not determined a priori as is the case for the CENSUS-X-11 procedure. Further difficulties in assessing the filter properties may emerge for some procedures due to non-linear operations. It is, however, possible to derive the necessary information of filter properties empirically. This is based upon the idea that the behavior of a linear filter is known for certain test signals. In this manner, it is also possible to recognize whether or not the procedural filter properties in the series center and at the boundary are different. For those procedures that have non-linear relationships, the derivation of the frequency domain properties with the aid of the theory of linear filters represents only an approximation. It may be demonstrated, however, that such an approximation (for example, in the case of the CENSUS-X-11 procedure) may produce useful information [32].

4. Designing Seasonal Filters (Transfer and Phase Functions)

The connotation of the term "filter" implies an algorithm used to transform a times series. The formal relationship between a time series to be filtered x_t, the so-called "filter-input", and the filtered series y_t, the so-called "filter-output", may be illustrated as follows:

$$y_t = F(x_t) .$$

For instance, moving averages or first differences are examples of simple commonly used filters.

The efficiency of a filter upon the filter-input may best be evaluated in the frequency domain applying a so-called "transfer function" T(f). Generally, this is a complex valued function, that may be stated as

$$T(f) = |T(f)|e^{i\phi(f)} , \quad 0 \leq f \leq 0.5 .$$

$|T(f)|$ designates the absolute value of T(f) - commonly called the *amplitude function* of the filter - (in the following stated briefly as A(f)), whereby $\phi(f)$ represents the respective *phase function*. i is the imaginary unit.

With the aid of the amplitude function it is possible to derive the properties of a filter in the frequency domain. Frequency components for which A(f) is smaller than one will be attenuated by filtering, whereas frequency components with A(f) greater than one will be amplified. Given A(f) = 1, the respective frequency components remain unchanged by the filter. If A(f) takes on the value zero for certain frequencies resp. frequency domains, then the corresponding components are fully eliminated.

The phase function $\phi(f)$ shows which *time lag* exists between filter-input and filter-output for which frequency components. $\phi(f)$ measures the phase in radians, which is not especially intuitive. Therefore a conversion into time units by dividing $\phi(f)$ by $f \in (0, 0.5]$ is advisable.

In this study, we mainly apply intensively two types of filters, namely low- and high pass filters.

An *ideal* low pass filter has the following amplitude function (see Figure 46):

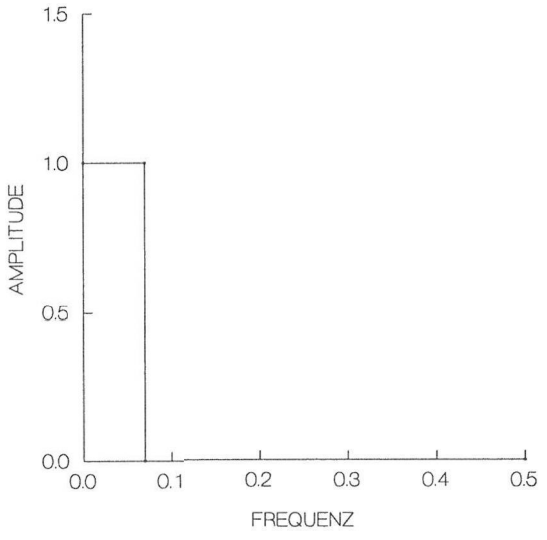


Figure 46

This filter does not affect oscillations with a frequency $f \in [0, f_A]$, but absorbs or eliminates oscillations with $f \in (f_A, 0.5]$. Hereby, f_A is the so-called "cut-off frequency", which in the above Figure was set at 0.07. Frequency domains that are brought forward by a filter are called *pass bands*^\, those that are eliminated are called *stop bands*. A low pass filter thus has a pass band $[0, f_A]$ and a stop band $(f_A, 0.5]$. This is illustrated in the Figure by the frequency domains (0, 0.07] resp. (0.07, 0.5]. An *ideal* low pass filter is thereby characterized by the fact that its amplitude in the pass band takes on *exactly* the value one and *exactly* the value zero in the stop band.

Figure 47 illustrates the amplitude function of an *ideal* high pass filter:

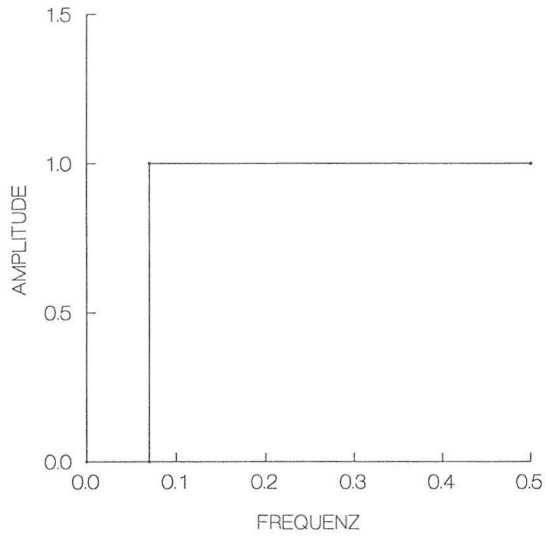


Figure 47

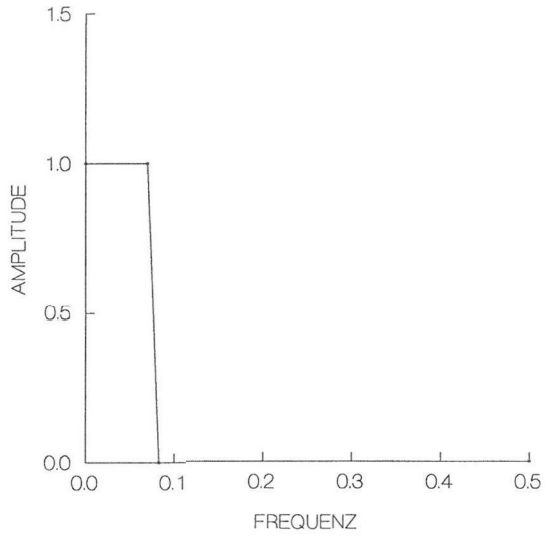


Figure 48

This filter does not modify oscillations with a frequency $f \in [f_A, 0.5]$ and eliminates all others. In the Figure $f_A = 0.07$ once again was chosen. In the stop- resp. pass-band the amplitude of this filter exhibits *exactly* the above prescribed values of zero and one, respectively. Obviously, the amplitude functions of low- and high pass filters are *complementary* to one another. Thus,

$$A_{HP}(f) = 1 - A_{TP}(f) .$$

The cut-off frequency f_A must be determined for both filters dependent upon the *objective* of the analysis. For instance, given a monthly time series, one desires to filter out all oscillations with a period length of 12 months or less, a low pass filter should be used for which $f_A < 1/12$. The resulting filter output corresponds to what is usually regarded to as the *smooth component*. If on the other hand all long run movements of a time series are to be eliminated, a high pass filter may be used (for instance, with the same cut-off frequency as the corresponding low pass filter). The filter-output then matches what is traditionally referred to as the *trend adjusted* series.

It should be noted here that simultaneously with the fixing of the cut-off frequency also the (intended) component definition of the concrete case at hand of the analysis is revealed.

Regarding the above filter as *ideal*, the presumption suggests itself that *real* filters do not exactly exhibit the presented amplitude functions. Without going into technical details, notice is taken here simply of the fact that the amplitude functions of real filters can not exhibit the above "rectangle form". Rather for specific reasons, a so-called "transition band" must always be implemented between the pass- and stop band, so that intended amplitude functions of low- and high-pass filters show a "trapezoidal" form, as may be seen in both of the following Figures 48 and 49 for which the length of the transition band is 0.01, respectively.

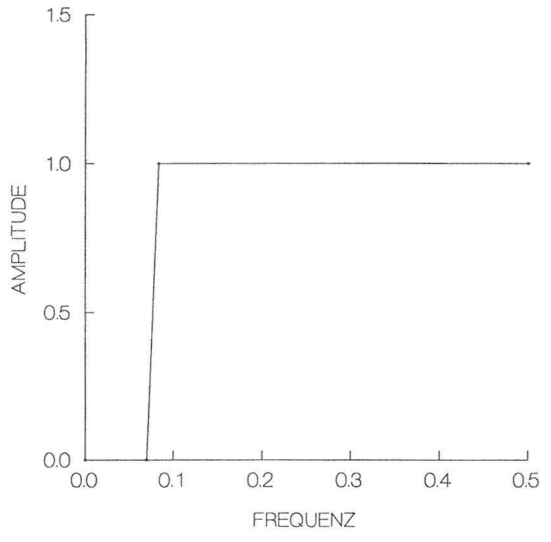


Abb. 49

One objective of *filter design* consists in constructing filters that approximate a given amplitude function "as close as possible". It is beyond the scope of this study to present here special methods of filter design. However, it should be mentioned that in designing filters also the corresponding generated phase must be taken into consideration. Of course, for practical purposes of time series analysis a zero-phase filter would be ideal. These may indeed be designed, however, with the disadvantage of having non-perfect boundary stability.

Figure 50 below illustrates the pattern of an ideal filter for *seasonal adjustment* of monthly time series.

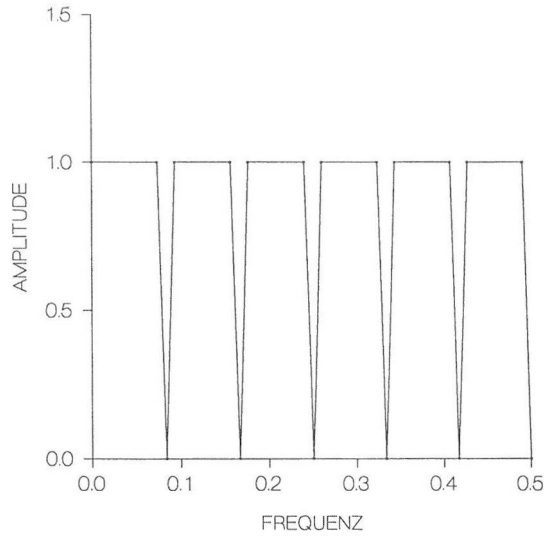


Abb. 50

This amplitude has the property that it has a zero value for the seasonal frequencies 1/12, 2/12, 3/12, 4/12, 5/12 and 6/12 and a value of one otherwise. The corresponding filter - known as the (multiple) notch filter - therefore affects only the seasonal component of a time series and leaves all other components unchanged. (For the adjustment of quarterly time series the corresponding amplitude functions exhibit only two notches at the frequencies 0.25 and 0.5).

Various methods exist to design filters that approximate these amplitude functions. For example, a so-called "IIR-Filter" can be constructed, whose amplitude differs only marginally from the ideal [33].

Seasonal adjustment with the aid of a filter of that kind goes beyond the so-called "classical decomposition model" that usually contains four components (trend-, cycle-, seasonal-, and residual (irregular) components). Instead, one postulates a priori an *infinite number* of components that correspond to a *continuum* of frequencies [0, 0.5]. It may, however, be demonstrated that seasonal adjustment procedures based upon traditional methods of analysis (usually moving averages and/or local regressions models) ultimately also attempt to realize the above mentioned ideal amplitude functions, although in an "indirect" way. This is especially the case for the CENSUS-X-11 procedure, as is obvious from the discussion presented in Chapter III. As such, this ideal amplitude function is very suitable as a *standard of reference*, for example, when the filter properties of various seasonal adjustment procedures are to be examined and compared.

5. Properties of Some Common Filters

5.1 Preliminary Remarks

During the discussion of trend elimination presented in Chapter I, Section 3.4 above it was pointed out, that in practice the method of first differences is commonly applied. It was also negatively remarked that this widespread practice can lead to serious distortions. In the following these are discussed and demonstrated by a few illustrative examples. Also, a critical analysis of an additional widespread filtering practice, namely that of taking annual differences, is reviewed critically. The following discussion of the most important relationships is again illustrated graphically, whereby the simulated series from Chapter I are referred to.

5.2 First Differences

Given the time series x_t, t = 1, 2, ... ,T and taking first differences the following transformation holds:

$$y_t := x_t - x_{t-1} .$$

Despite the fact that the pure sinus wave observed in Chapter I, Section 3.1 above does not exhibit a trend and therefore there is no necessity whatsoever for differencing, let us all the same first of all examine the consequences of taking differences on this time series. Figure 51 illustrates both the original and the filtered series.

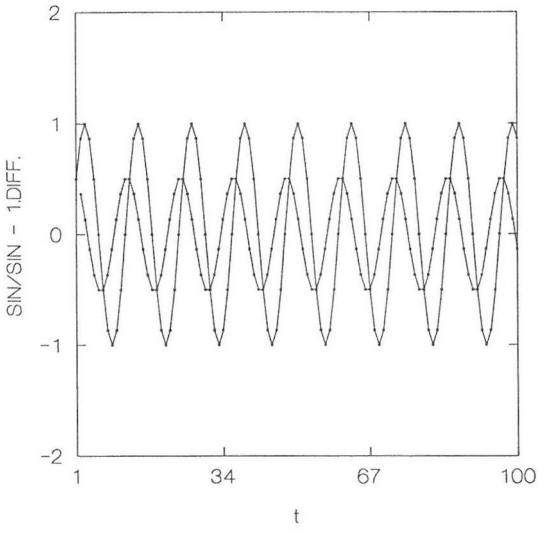


Figure 51

Two consequences of the difference-filter may be immediately observed. Firstly, the filter attenuated the amplitude of the sinus wave. Secondly, the filter-output exhibits a phase shift in relation to the filter-input, i.e. compared to the original series. Both of these effects are to be found given arbitrary input signals, as will be demonstrated below.

The attenuation of the amplitude may be seen quite clearly in the spectrum of the filtered series. Figure 52 contains the spectra of both series, the "peak-ordinate" is substantially smaller for the differenced series.

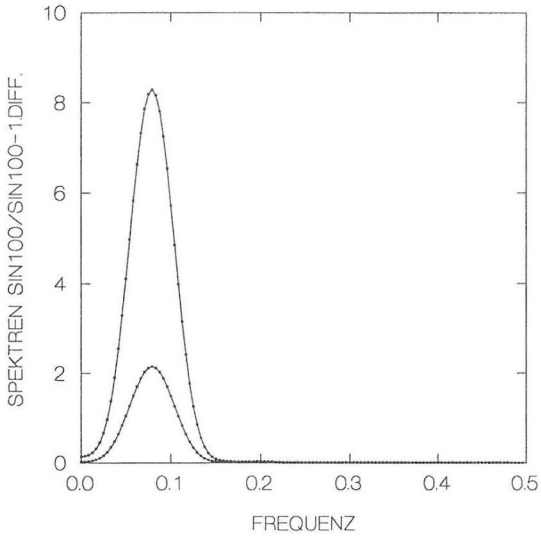


Figure 52

If one takes first differences of the series from Figure 25 in Chapter I, Section 3.4 (trend + sinus wave) and compares these with the sinus wave, Figure 53 is obtained:

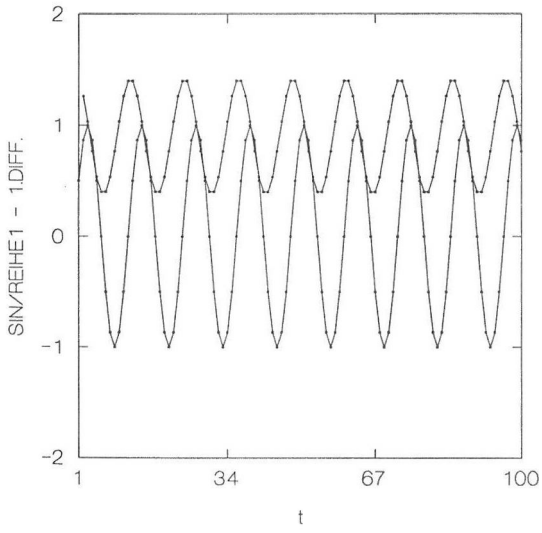


Abb. 53

In addition to the attenuated sinus oscillation and a phase shift, one also observes a parallel shift of the filter-output. This appears quite reasonable if the filter-ouput is calculated, which may be simply demonstrated. One obtains

$$y_t = 2+0.9t + \sin(2\pi/12t) - \{2+0.9(t-1)+\sin(2\pi/12(t-1)\}$$
$$= 0.9+0.134\sin(2\pi/12t) - 0.5\cos(2\pi/12t) ,$$

i.e. by taking differences, a positive constant remains from the linear trend which basically induces the shift. First differences imply not only attenuations, but also *amplification* effects. This is clearly demonstrated by the following example in Figure 54 below, in which first differences were taken for series 4 from Figure 31 above.

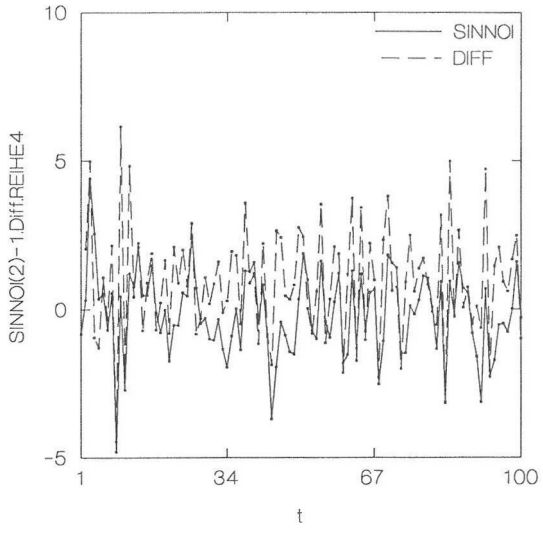


Figure 54

It is clearly observable that the variance of the differenced series (dashed curve) is significantly larger than the variance of the "signal"

$$\sin(2\pi/12t) + \varepsilon_t ,$$

which is to be expected by an exact elimination of the trend (3.5 as compared to 2.1). Taking differences brings about an increase in the variance of 40%.

Figure 55 below shows the estimated spectra of the signal, of the high pass, as well as the series filtered by differences.

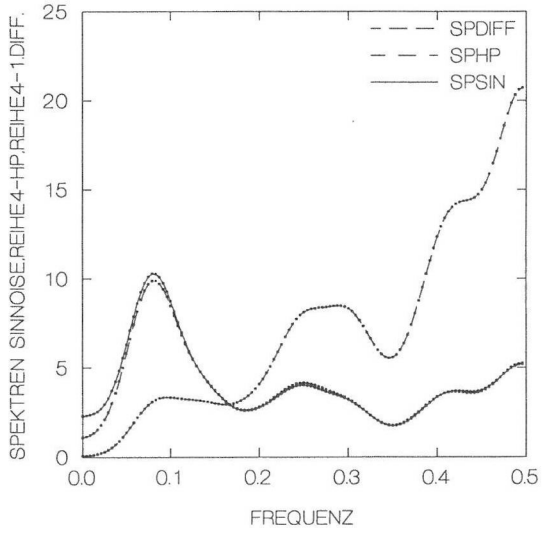


Figure 55

The strong shift of spectral mass into the high frequency domain by the difference filter is clearly evident, which explains the erratic path of the first differences. Further-

more, the sinus oscillation is so strongly attenuated that it no longer shows up in the spectrum.

The above demonstrated effects of taking differences may generally also be detected for the corresponding amplitude function. The equation for the amplitude function is:

$$A(f) = [2(1 - \cos 2\pi f)]^{1/2} \ , \ f \in [0, 0.5] \ .$$

The graph of this function is displayed below in Figure 56.

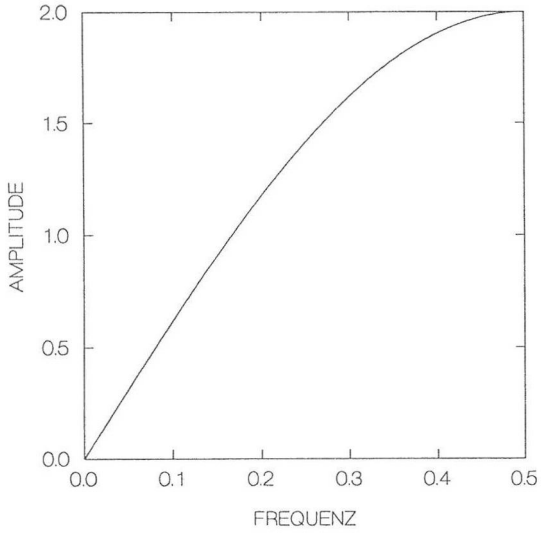


Figure 56

The amplitude is smaller than one up to the frequency f = 0.2 (oscillations in the frequency domain 0 - 0.2 are more or less attenuated). Thereafter, the amplitude increases monotonically till it reaches a value of two, which causes a shift of spectral mass into the high frequency domain. The consequence being an (undesired) inflation and amplification of the variance. Since the amplitude at f = 0 has the value zero, linear trends are eliminated.

In summary, the following conclusion holds:

The common practice of trend elimination by first differences is an operation that is characterized by serious disadvantages (undesired attenuations resp. amplifications and phase shifts). Therefore, for the elimination of trends, procedures should be applied that avoid such disadvantages. Suitable filters have been designed taking into account prespecified criteria.

5.3 Annual Differences

Annual differences are commonly used in practical applications of seasonal adjustment. The amplitude of this difference filter may be stated as

$$A(f) = [2(1 - \cos 24\pi f)]^{1/2} \ , \ f \in [0, 0.5]$$

and exibits the following path (see Figure 57):

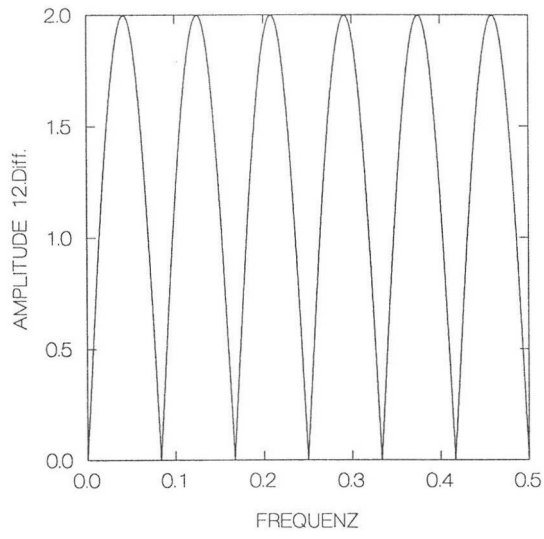


Figure 57

For the frequency f = 0 the function equals zero, as also is the case for *all* seasonal frequencies f = 1/12, 2/12, 3/12, 4/12, 5/12, 6/12. As such, this filter appears to be appropriate for a trend adjustment as well as a seasonal adjustment. However, as may be easily shown, taking annual differences is a rather poor seasonal filter. For example, applying this procedure to the following series (see Figure 58),

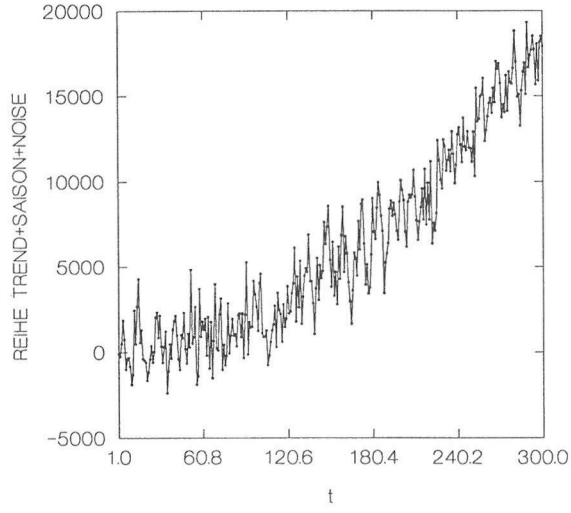


Figure 58

which is based upon the model

$$X_t = 2 + 0.9t + 0.2t^2 + 1000 \ (\sin 2\pi/12t + \varepsilon_t) \ ,$$

$$t = 1,...,300, \text{ mit } \varepsilon_t \sim N \ (0, 2) \ ,$$

one obtains the filter-output displayed in Figure 59:

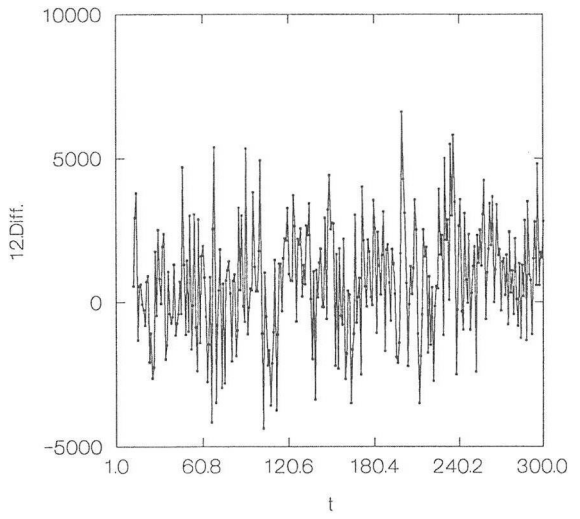


Figure 59

That the seasonal component is not entirely filtered out of the series may be seen by looking at the estimated autocorrelation function, which exhibits a value significantly different from zero at lag 12 (ref. to Figure 60):

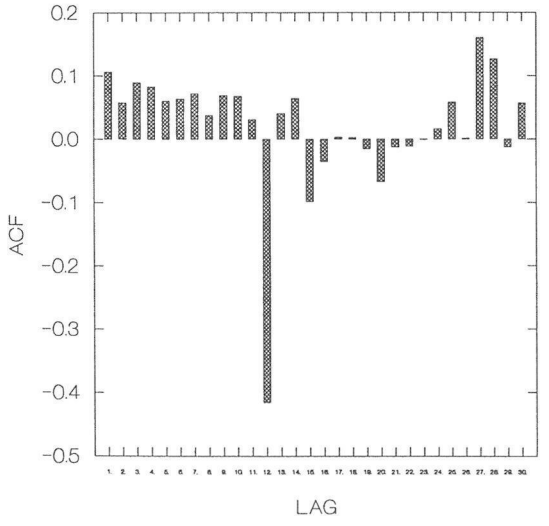


Figure 60

Furthermore, the trend removal of the above series is also not completely satisfactory. As may easily be derived based upon the component model applied, after taking the differences not only a constant term is left over, but also the linear term 4.8t remains. If one estimates for the filter-output a linear regression with time as the independent variable, then one does indeed obtain an estimated value of 4.43 which is significantly different from zero.

As is easily reflected, both first differences as well as annual differences are only capable of filtering out *linear* trends.

Just as first differences increases the noise of a series, the same holds for annual differences. The above mentioned amplifications occur also. Whereas the variance of the component

$$1000(\sin 2\pi/12t + \varepsilon_t) \ , \ t = 1,...,300$$

amounts to 2'348'160, the value of the annual differences equals 3'974'740, representing a variance inflation of about circa 69%.

It is not uncommon in practice that a series is adjusted by both first differences and annual differences. Taking first differences removes the trend and taking annual differences causes a seasonal adjustment. Although a quadratic trend may be filtered out by so doing, such a method has even more devasting disadvantages than those already discussed. The amplitude function of a combined first and annual difference procedure is given as

$$A(f) = [2(1 - \cos 2\pi f)]^{1/2}[2(1 - \cos 24\pi f)]^{1/2}, \ f \in [0,0.5]$$

and displays the following path (compare Figure 61):

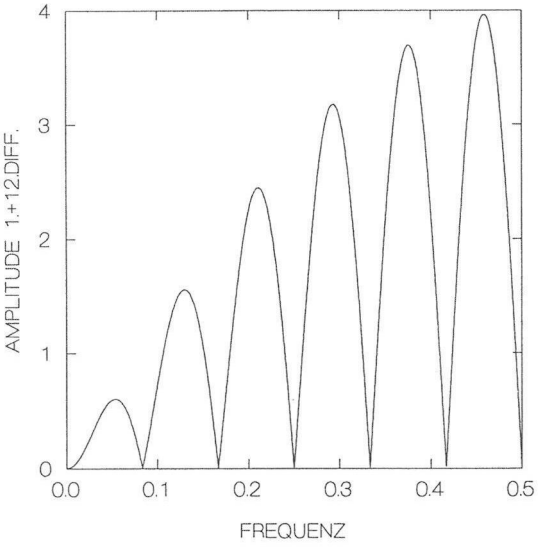


Figure 61

Due to first differences, attenuations are to be observed in the low frequency domain. In the high frequency domain one also observes amplifications which are now even stronger than given the case of just one filter alone.

Constructing first and annual differences for the above series one obtains Figure 62 below:

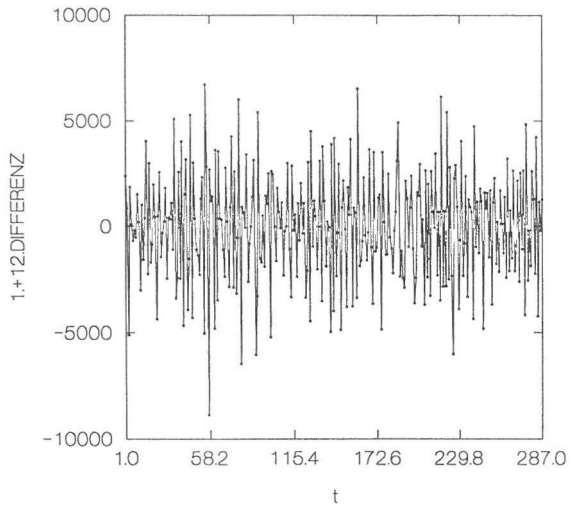


Abb. 62

The variance of this filter-output amounts to 7'115'820, which represents an inflation of 203% (!) compared to the variance of the "signal" and an inflation of 79% compared to a pure annual difference.

The amplitude functions and the above discussed examples show vividly that taking differences is indeed not a harmless operation. It must especially be concluded that differences are in general inappropriate tools for trend removal and seasonal adjustment.

5.4 Growth Rates

Finally, we consider a special, but frequently used time series filter that is closely related to the procedure of taking differences. This filter uses *growth rates*, whereby the reference period usually chosen is the previous period:

$$w_t = \frac{x_t - x_{t-1}}{x_{t-1}} .$$

By calculating growth rates one hopes to obtain information on the current tendency of a series (and thereby also on the underlying economic process). The question arises, whether or not it is generally possible by the simple comparison of *only two* observations to determine the tendency of the process. The answer to the above stated question is a definitive *no* for the following reason.

It is indeed correct to say that monthly growth rates contain the current tendency of interest. However, this information is hidden by the growth rates of the remaining components, especially by the noise component and the seasonal component. The occurence of possible outliers accentuates this matter of fact. Therefore, growth rates based upon the previous period are of little relevance with regard to the current tendency of a series.

The following presents and explains the above argument in detail.

With regard to the above statement, let us start with the simulated series from Figure 63 below, which is founded upon the following component model:

$$X_t = 2 + 0.9t + \varepsilon_t \quad , \quad t = 1,2,...,100, \text{ with } \varepsilon_t \sim N(0,2)$$

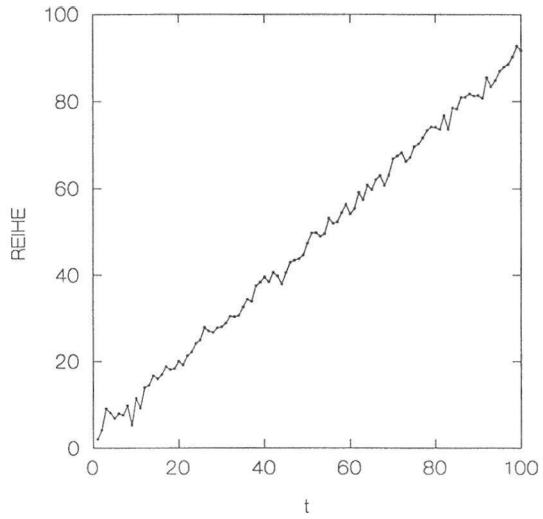


Figure 63

This series may be characterized as perfectly "harmless" and exhibits a clear tendency, which is of course determined by the linear trend. Does this tendency show up in the growth rates of the series, i.e. in the comparison of two observations?

Figure 64 below illustrates the growth rates of this series.

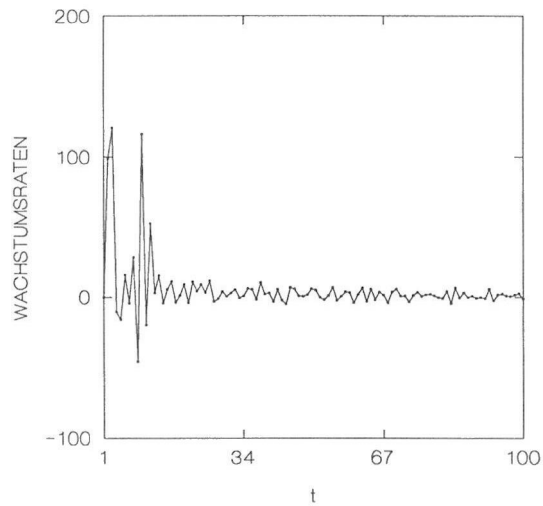


Figure 64

Obviously, growth rates supply neither information about the general nor about the current tendency of this series, for they exhibit a range from -45.765% to 120.96% with a variance of 440.7! They must therefore be characterized as

fully misleading, since a linear trend does not exhibit negative growth rates. The respective formula is

$$WR_{Trend} = \frac{0.9}{1.1 + 0.9t}$$

and displays a monotonically decreasing (but always positive) path as illustrated below in Figure 65:

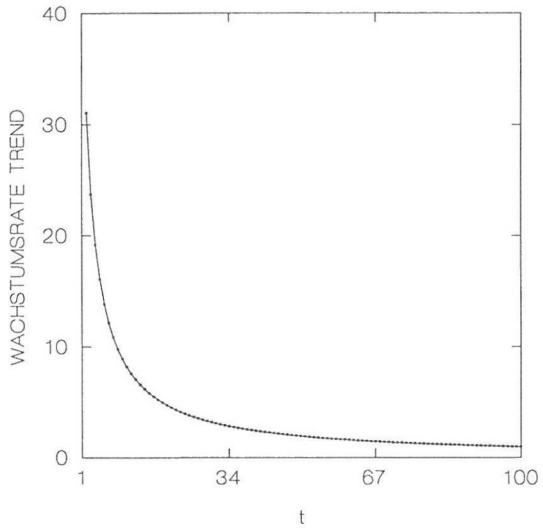


Figure 65

These (real) growth rates lie within a range of 0.99% and 31% with a variance of 22.4. In this case, entirely different magnitudes for the rates are given compared to those derived previously from the series.

The cause of the observed erratic behavior of the growth rates is due to the fact that period growth rates are nonlinear filters. They feature approximately similar filter properties as those of first differences, which were previously presented above.

In order to obtain reliable information regarding the tendency of a series, more than two observations must be considered. For instance, in the above example, the noise must be filtered out, which may be accomplished by a low pass filter taking account of all observations. An exact low pass filter basically reproduces the linear trend as the correct tendency. The corresponding growth rates calculated deviate only insignificantly from the actual rates.

Figure 66 jointly illustrates the real growth rates (i.e. of the linear trend) and the growth rates of the series along with the growth rates based upon a low pass filter.

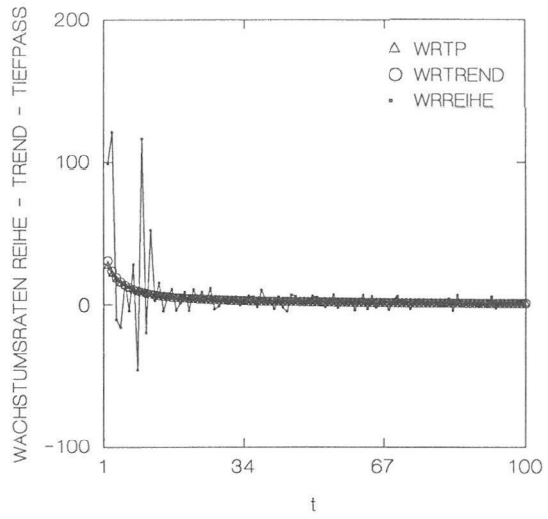


Figure 66

Finally, let us take a look at an especially lucid example of misleading information with regard to the tendency of a series based upon growth rates. The series in Figure 67 is generated by the following model:

$$X_t = 0.5e^{0.05t} + \varepsilon_t \quad , \quad t = 1,2,...,100, \text{ with } \varepsilon_t \sim N(0,2)$$

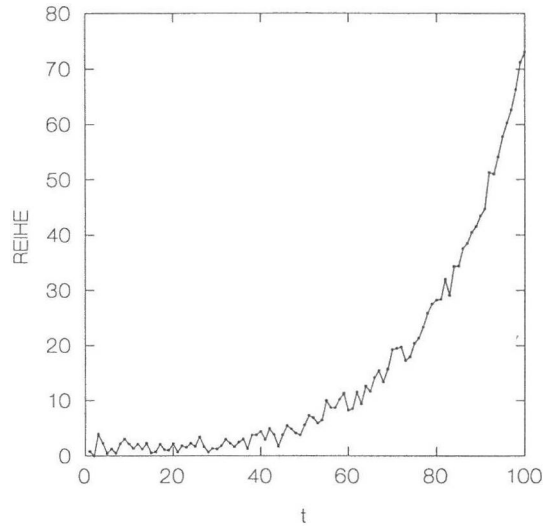


Figure 67

The development of this series is determined by an exponential function. This function increases over the entire time domain with a *constant* growth rate of 5.13%. The growth rates that are calculated from the series vary, however, in the interval from -99.2% to 58,441% with a variance of 34,137,500! This is a practically absolute irrelevant, if not absurd result.

In summary, it must be concluded that the widely held idea that by comparing neighboring observations information on the tendency of a series may be obtained, is not sensible.

In practice, one commonly finds in addition to simple growth rates also growth rates based upon the observations of the previous year, e.g. October 1992 in comparison to October 1991. It may easily be demonstrated that such yearly comparisons should also be judged just as negatively as the above considered growth rates.

Let us consider a series composed additively of the last series and the seasonal figure from Figure 1. The magnitude of the seasonal figure as compared to the pronounced trend is of a fully subordinated importance. As illustrated below in Figure 68, consequently, this seasonal series (mean: 15.04, variance: 343.3) hardly differs from the previous (non-seasonal) series (mean: 15.06, variance: 347.7).

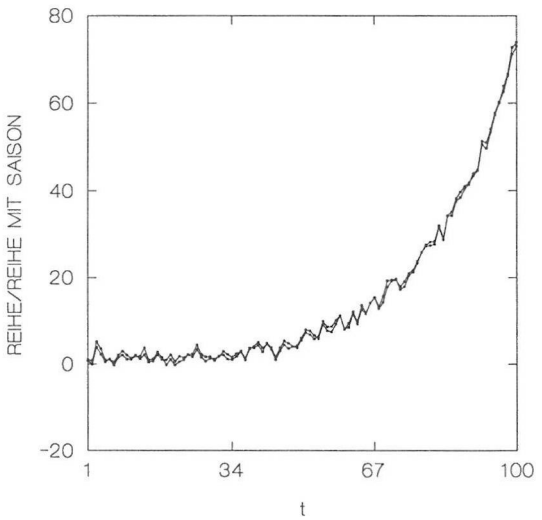


Figure 68

Annual growth rates are shown in Figure 69:

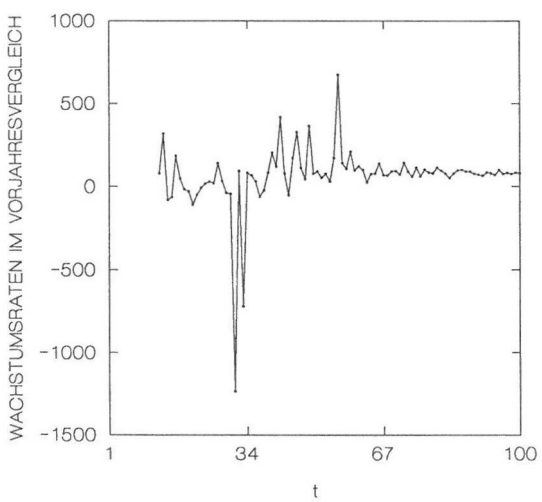


Figure 69

These growth rates exhibit a range from -1.234.7% to 672.4% with a variance of 37.839.5. This result is just as absurd as the previous one. Yearly growth rates therefore supply as little information about the tendency of this series as did growth rates based upon the previous period.

One may object to these examples on the grounds that the very extreme fluctuations of the grow rates are above all contingent upon the small observations at the beginning of the series and that this situation is not very typical for actual economic time series. In particular, for these values the noise dominates. This objection is to a certain extent justified; however, it does not fundamentally change the above criticism, even though it may weaken it more or less for specific time series at hand. For instance, given a series with little noise and only small seasonal fluctuations, resp. possessing a rather rigid seasonal figure, then deriving the growth rates does not lead to the above demonstrated extreme results. However, given economic series such conditions are generally not fulfilled to any needed extent so that working with growth rates will usually bring about substantial distortions, making any predictions of the series development at least questionable.

6. Seasonal Filter Construction Problems

At the forefront of the discussion today, are above all the properties of the transfer function as defined by the filter of the respective seasonal adjustment procedures. In the literature general consensus holds with regard to which transfer function should be considered "ideal" for seasonal adjustment. The ideal transfer function is characterized by the fact that only seasonal frequencies (e.g. measured in cycles/TU or in radian) are affected, whereas all other frequencies resp. frequency domains, in particular, however, the frequency domain below the first seasonal frequency (e.g. given monthly values the interval [0, 1/12)), should remain unaffected. This means that it is required of the transfer function that the corresponding seasonal filter has the property of eliminating only the variation of the time series that may indeed be interpreted as seasonal. Two important restrictions must be further observed given the design problem of a seasonal filter. First, the filtered series should be as stable as possible at the boundary. Second, a phase shift between the adjusted series and the observed series should not occur or if it occurs it should be negligible, i.e. its magnitude should be smaller than the basic time period (zero phase). Numerous evaluation criteria are conceivable in order to judge the goodness-of-fit of the approximation of the desired transfer function by the realized one. The solution of the design problem is, however, quite an intricate and by no means trivial problem. Yet the approximation of an ideal transfer function and the fulfillment of the above restrictions do yield useful and operational criteria for judging the quality of alternative seasonal adjustment procedures.

Looking at the entire discussion with regard to evaluation criteria of seasonal adjustment procedures to date, it is quite astonishing that seldomly the problem is discussed that seasonal adjustment procedures do not predominately apply

final filters. The deviations between an adjustment up to the boundary and an ex post adjustment - apart from the question of boundary stability - have been barely discussed in practice. The evaluation criteria have dealt mainly with ex post adjustments, the properties of final filters were generally left out of any discussion with the exception of the above mentioned stability problem. Nevertheless as a rule, seasonal adjustment is usually pursued in analysing current situations and not primarily for purposes of economic history. On this aspect, however, the properties of final filters should above all be of interest.

It is easy to comprehend that an exact amplitude function, an exact zero phase and perfect boundary stability may not be realized simultaneously. Perfect boundary stability may only be obtained given a filter that is causal (or recursive). The filter equation for such a filter of order (N,M) is:

$$y_t = \sum_{k=1}^{N} a_k y_{t-k} + \sum_{j=0}^{M} b_j x_{t-j}, \quad t = 1,...,T$$

with x_t : = filter input (i.e. a given time series),

y_t : = filter output,

a_k, b_j : = coefficients based on the type of filter.

It may be directly observed from the filter equation that previously filtered values no longer change if new observations are added, consequently therefore perfect boundary stability is given. The frequency - response function of such a filter - which is relatively simple to determine using a z-transformation of the above filter equation (for values on the unit circle) - shows that this function is always a complex, rationale function in the frequency domain [0, 1/2]. The absolute value of this function is the amplitude function of the filter and the arc tan of the ratio of the imaginary and real part of this function is the corresponding phase function. This phase function is in principle nonlinear. Accordingly, a filter with perfect boundary stability cannot have an exact zero phase.

It may further be shown, that generally a certain antinomy between the amplitude function and the phase function exists. The more focused the amplitude function is (given the ideal case: exactly equal to zero in stop bands and exactly one in pass bands), the larger are the values taken by the phase function.

In the following table to begin with, various types of filters are listed, categorized according to whether or not filtering is done in the time domain or in the frequency domain. Given filters associated with the first type, we differentiate between ad hoc filters, FIR-filters and IIR-filters [34].

		Realizable Properties Of Filter Types					
Filtering in:	**Filter Type**	**Exact Amplitude Function**	**Exact Zero Phase**	**Perfect Boundary Stability**	**Known Procedures For The Respective Filter Type**	**Finality Of The Applied Filters**	**Usage Of Boundary Filters**
Time Domain	ad hoc construction	no	to some extent, often difficult to control	no	CENSUS X-11, BERLIN METHOD, ASA II, SABL	not present	yes
Time Domain	FIR	no	yes**)	-***)	none		
Time Domain	simple IIR	no	no	yes	none		
Time Domain	optimal IIR	no	no	yes	none		
Time Domain	direct IIR	no*)	no*)	yes	STIER	present	no
Time Domain	special IIR	no	no*)	yes	SCHIPS/ STIER	present	no
Frequency Domain	null phase	yes	yes	no*)	SCHMIDT/ STIER	not present	yes

*) Theoretically strictly correct, however, also obtainable with sufficient exactness in practice.

**) Only for symmetric implementation.

***) Has no meaning for this type of filter, since filtering up to the boundary is not possible.

Column 6 in the above Table presents a listing of procedures and their respective types of filters, whereby no claim of completeness is made [35].

The problem of whether the filter used in a given procedure takes into account the most recent time series value is addressed in column 7 above. Filters featuring this characteristic, as mentioned earlier, are called "final" filters. If a given filter is not a final filter, then it must be supplemented with a boundary filter, otherwise an adjustment of the most recent obeservations is not possible. Determining from which time point on a "boundary filter" becomes necessary, varies of course from procedure to procedure. It is quite common that various versions of same procedure differ, among other things, largely by the time point from which on the boundary filter is implemented or with regard to the design of the resp. boundary filter. For instance, in Version 1 of the BERLIN METHOD a boundary filter is used for estimating the smooth component from time point n-9 on (n: last observation). Version 2 modifies this value to n-11. Furthermore, the resp. boundary filters are designed differently [36]. Procedures that implement final filters are also characterized by perfect boundary stability.

In addition to the above presented filter-theoretical principles, it is also essential in designing and evaluating seasonal adjustment procedures that one considers certain methodological postulates [37]. In empirical research nowadays, these postulates are taken more or less for granted. Unfortunately, however, with regard to principles of the philosophy of science there still often exists a lack of general understanding of these points in the context of statistics. These aspects should no longer be outside the relevant basis of discussion. It is surely not an exaggeration here to state the opinion that the often seemingly chaotic discussions found dealing with problems of seasonal adjustment are to a large extent based upon the fact that methodological aspects are simply ignored. This has by no means anything to do with a need to take into consideration any "esoteric dimensions". Rather, it has to do basically with paying due attention to certain methodological aspects that should fundamentally be quite obvious.

First of all, it can be stipulated that "research" generally may be presented as a sequence of decisions. Each individual research project, as for example the design of a seasonal adjustment procedure, implies an abundance of decisions that may have large efects upon the derived research results. It is therefore necessary to carefully document such decision processes in order to guarantee the possibilty of intersubjective controllability, a demand that has also been raised by critical rationalists. The above statement makes it essential for researchers to confront themselves with problems of concepts, definitions, as well as operationalization strategies independent of any concrete adjustment procedure.

Practical application of concepts in research necessitates that they are clearly defined. Demanding intersubjective comparison of seasonal adjustment procedures compels the researcher to be open for discussion with regard to definition problems of concepts. In general, one may essentially differentiate between "nominal" and "real" definitions. Nominal definitions are no more than tautological statements. The "definiendum" is synonymous for the "definiens". Such definitions have no empirical content. They basically aid and allow an intersubjective control of a research process, since only what is defined in a precise way can be checked unequivocally. In contrast, real definitions portray the "essence" of the matter at the linguistic level. They represent propositions about the nature of a phenomenon and thus have the same status as empirical hypotheses and as such they may be right or wrong. However, as a rule, real definitions may also be incomplete, since it is usually impossible to include all properties of a phenomenon within a single definition. But the purpose and objective of real definitions is exactly to bring only the "essentials" to one's attention. The problem remains, however, that it is impossible to determine in a perfect fashion what is "essential" or "inessential". In the past, the question of which of the above definitions is to be preferred was at the center of intense controversy in the philosophy of science. For practical research applications, however, such debates are generally not productive, since one is usually left with choosing a compromise between both approaches. Basically, therefore a researcher attempts to establish what are the "essentials" of the phenomenon and applies these essentials in the sense of a nominal definition in all further considerations.

Surveying the underlying definitions of "seasonality" in various procedures, one generally finds elements of such "nominalized" real definitions or nominal definitions supplemented by features of real definitions. The reason for finding only elements is that the revealed definitions are often quite vague, so that in a strict sense they do not represent definitions at all. Usually, one is confronted with purely verbal characterizations of the phenomenon "seasonality". This does of course not imply that even though a procedure may not possess a clear, distinct and comparable definition of "seasonality", such a procedure operates without a definition at all. On the contrary, such a procedure uses an implicit definition, provided by its program steps which may, however, not be ascertained.

Consequently, we can state a first evaluation criterion:

- Seasonal adjustment procedures must explicitly be formulated using a precise, clear definition of "seasonality".

Precise definitions are, however, only a necessary but not sufficient condition for correct methodological research. Since definitions are created on a linguistic level only, it is not possible with their help alone to accomplish the necessary link between concepts and real phenomena. This can be achieved only by operationalization. Operationalization is used here to imply the statement or determination of

the particular procedural steps (or research operations) which are employed to decide whether or not and to what extent the concept and phenomenon alluded to is present in reality. Thus, after a precise definition of "seasonality" is given, the explicit technical operations must be presented that permit an evaluation on whether or not and to what extent a series is influenced by "seasonality" (in the defined sense). These could be for example: design a filter with specified properties or optimize an objective function with exactly specified properties etc. One must thereby always keep in mind that any chosen operationalization must be logically valid, that is it must be checked whether or not the applied definition and operationalization are of equivalent intrinsic value. In addition to the logical contents of an operationalization one must also examine its empirical validity. In this context it must be demonstrated that the operationalization does indeed "grasp" reality as theoretically defined. It is not necessary that logical and empirical vailidity holds simultaneously. For instance, a theoretical specification of a filter may be 100% valid logically, but the realized filter may deviate somewhat from its theoretical ideal pattern due to manifold reasons, therefore being not 100% valid empirically. Given the above considerations, three further evaluation criteria emerge:

- The operationalization of "seasonality" must be clearly recognizable for seasonal adjustment procedures.

- Seasonal adjustment procedures must be designed so as to permit an evaluation as to whether or not and to what extent the operationalization of "seasonality" is logically valid.

- Seasonal adjustment procedures must be designed so as to permit an evaluation as to whether or not and to what extent the operationalization of "seasonality" is empirically valid.

Directly linked with the criteria of logical validity is a matter of fact that is fundamental for some procedures. Namely, that these procedures offer certain user options, e.g. additional smoothing of the adjusted series, whereby the user can determine the degree of smoothing by choosing the value of a parameter within certain limits. Attention must thereby be addressed to the question of whether or not the turning of certain "screws" is still compatible with the chosen definition of "seasonality". This leads to the next criterion:

- Given seasonal adjustment procedures equiped with user options, it must be acertainable whether or not any alternative options chosen are compatible with the originally chosen definition of seasonality.

The following criterion should be self-evident:

- The intended objectives of the seasonal adjustment procedure should be obvious. Care must especially be taken to show that the chosen operationalization is compatible with these objectives.

Some adjustment procedures are constructed iteratively, that is to say that certain phases are processed several times over. This easily results in "pragmatic circles". If the smooth component is to be determined for instance, then the seasonal- and residual component must be determined also. But in order to determine the seasonal component one must first - although only preliminary - calculate the smooth component. Important in this regard is the fact that the preliminary "phases" of analysis prejudice the final estimation in an uncontrollable manner. Therefore, the following additional criterion is necessary:

- In designing seasonal adjustment procedures, it is necessary to avoid pragmatic circles.

The above criteria appear to be the most important elements in the present context of methodological principles that should be adhered to. In particular, the criteria dealing with definitions, operationalization and objectives must be regarded as essential. Since we make no claim of completeness here, it is of course conceivable that additional criteria are also relevant. Let it suffice here only to mention one more criterion that should be observed, especially when comparing procedures. This criterion results from the fact that given the specific procedures a number of hypotheses are postulated which, as a rule, may not be empirically tested. Consequently, a procedure that contains more nontestable hypotheses in comparison to another procedure is -ceteris paribus - to be regarded as inferior. Thus, the last criterion may be formulated as follows:

- Considering methodological aspects in evaluating two seasonal adjustment procedures, a respective procedure is regarded as inferior - ceteris paribus - when it contains a larger number of non-testable hypotheses.

The following table presents a selection of important methodological postulates.

Procedure	Evidence of the Procedural Objective	Precise Definition of "Seasonality" Present	Recognizable Unique Operation-alization of "Seasonality"	Existence of Pragmatic Circles
CENSUS X 11	no	no	no	yes
BERLIN METHOD	?	?	?	yes
ASA	no	no	no	yes
SABL	no	no	no	yes
STIER	yes	yes	yes	no
SCHIPS / STIER	yes	yes	yes	no
SCHMIDT / STIER	yes	yes	yes	no

7. Design of Low Pass Filters

7.1 Filters in the Time Domain: FIR-Filter and IIR-Filter

In many problems of time series analysis such filters play an important role, for which the relationship between filter input and filter ouput takes on the following form of a linear difference equation:

$$y_t + a_1 y_{t-1} + \ldots + a_M y_{t-M} = b_0 x_t + b_1 x_{t-1} + \ldots + b_N x_{t-N} .$$

Obviously, one is dealing here with causal (or non-anticipatory) filters, since the filter output is dependent not only upon earlier output values but also upon current inputs and previous inputs. It may easily be demonstrated that the transfer function of such a filter is a rational function that is exclusively determined by the "filter coefficients" $a_1, a_2, \ldots, a_M, b_1, b_2, \ldots, b_N$.

Concerning the transfer function of such a filter resp. filter equation, two fundamental filter types may be distinguished. Given $a_1 = a_2 = \ldots = a_M = 0$, the transfer function displays only roots, but no poles. The filter output is independent of the output of previous time points. Such filters are designated as *FIR* (=*F*inite-*I*mpulse-*R*esponse)-Filters. Synonymous herefore is the terminology *transversal filter* and *non-recursive* filter.

The filter equation for a FIR-filter is then

$$Y_t = \sum_{r=0}^{N} b_r x_{t-r} .$$

In constrast, filters that also have poles besides roots, the coefficients $a_1, a_2, \ldots, a_M$ are not all equal to zero. Such filters are called *IIR* = (*I*nfinite-*I*mpulse-*R*esponse)-filters. With regard to their filter equation, the name *recursive* filter is commonly used.

A FIR-filter is specified when the filter coefficients b_0, $b_1, \ldots, b_N$ are defined. For example, a simple FIR-filter may be determined by choosing the filter coefficients arbitrarily. One obvious possibility is, for example,

$$b_0 = b_1 = b_2 = \ldots = b_N = \frac{1}{N+1} ,$$

i.e. this filter is a simple *moving average of the length (or order) N + 1* with

$$\sum_{r=0}^{N} b_r = 1 .$$

Accordingly, the well known simple moving averages (e.g. of length N = 12) are FIR-filter. Further, it may be demonstrated that these filters are also low pass filters.

The term *filter-design* is used in situations in which the filter coefficients are not determined ad hoc (such as is the case with simple moving averages), but rather are derived under consideration of specified design-principles. The coefficients are thereby generally specified so as to approximate "best" a target-amplitude and/or target-phase function. Due to lack of space, we avoid presenting here methods of filter design for FIR-filters [38].

FIR-filters and accordingly also the FIR-low pass-filters have the favorable property of exhibiting linear phases. Given that they are implemented symmetrically, they display *zero phases*. Unfortunately, an analysis up to the boundary is not possible using them. A further disadvantage of FIR-filters, however, is in general that "focused" amplitude functions may only be obtained for relatively large *filter orders* N, which given short time series as one commonly finds in practice often results in non-realization of the defined filters.

The filter equation for a IIR-filter may be expressed as:

$$\sum_{k=0}^{M} a_k y_{t-k} = \sum_{r=0}^{N} b_r x_{t-r}, \quad a_0 = 1 .$$

Since transfer functions of IIR-filters, in contrast to those of FIR-filters, have roots besides poles, additional *degrees of freedom* with respect to the filter design are given. The frequency-response function resp. amplitude function may not only be *influenced* by the placement of the roots (which necessitates proper consideration of *domain-specific requirements*), but also by the placement of poles in the complex plane. This possibilty increases the flexibility of filter design enormously. In particular, it is possible by adept placement of the poles given even small filter orders (i.e. small M and N) to obtain very sharp amplitude functions and also especially focused low pass filters.

For M = N = 1 one obtains e.g. the following filter equation:

$$y_t = b_0 x_t - a_1 y_{t-1}$$

resp.

$$y_t = \beta x_t + \alpha y_{t-1}$$

with

$$\beta := b_0 \ , \ \alpha := -a_1 .$$

It can be shown that this IIR-filter is a low pass filter if the parameter α is chosen to lie in the interval (0,1). For parameter values that are very close to one, one obtains low pass filters whose amplitude functions are more focused than those above of simple FIR-low pass-filters.

The advantage of greater flexibility in designing filters is however, also combined with a disadvantage. IIR-filters always exhibit a *non-linear* phase. Consequently, the design of zero phase IIR-filters is in principle not possible. Sometimes however, it is often the case that the phase-shift between filter-input and filter-output is small (e.g. two or three time units), which may be unproblematical for certain applications. However, if one irrevocably demands an exact zero phase, then IIR-filters may not be used. A detailed discussion of design methods for IIR-filters and IIR-low pass - filters in particular is not possible here. The interested reader should consult the above cited literature.

7.2 Filters in the Frequency Domain

Characteristic of the filters considered so far is that the filter design was formulated and solved in the *frequency domain.* But filtering is carried out in the time domain via the filter equation. Both FIR- and IIR filters demonstrated clearly the antinomy between amplitude and phase. Linear (and thus also zero-) phase is possible only given FIR-filters, focused amplitudes, however, require IIR-filters. It is therefore not possible to realize both desireable properties simultaneously, at least not with the design approaches discussed so far.

However, this is possible if filtering of a time series (or a signal) is carried out in the frequency domain. Since signals always appear in the time domain, filtering in the frequency- domain is only possible after transforming them into the frequency-domain. This can be accomplished by using the technique of the so-called "Discrete Fourier Transform" (DFT).

For a given signal x_t, t = 0,1,...,T-1 of length T, the DFT of this signal is defined by:

$$DFT(x_t) = X(e^{i\lambda}) := \sum_{t=0}^{T-1} x_t e^{-i\lambda t}, \quad \lambda \in [0,2\pi]$$

$X(e^{i\lambda})$ represents the complete frequency information of the signal x_t. Since $X(e^{i\lambda})$ is defined for a continuum of frequencies, a direct evaluation does not appear possible. Furthermore, a reproduction of the orignal signal from $X(e^{i\lambda})$ with the aid of an inverse transformation also appears to be impossible. It may, however, easily be shown that knowledge of $X(e^{i\lambda})$ for only T frequencies is sufficient in order to derive the original signal by means of a so-called "Inverse Discrete Fourier Transformation" (IDFT).

The following equation,

$$X_m = \sum_{k=0}^{T-1} x_k e^{-i(2\pi/T)km}, \quad m = 0,1,...,T-1$$

represents the DFT of x_t for the frequencies 0, $2\pi/T$, $4\pi/T$,..., $(T-1)2\pi/T$. The complex series $X_0, X_1, ..., X_{T-1}$ displays the transformation of the signal $x_0,...,x_{T-1}$ into the frequency domain. The IDFT of X_m is

$$IDFT(x_m): = \frac{1}{T} \sum_{m=0}^{T-1} X_m e^{i(2\pi/T)mt}, \quad t = 0,1,...,T-1 .$$

In practice, especially for small T's, it is common to pursue a "refinement" of DFT. Given small T's, the investigation of the frequency domain may prove to be too clumsy.

Now defining a signal by

$$\tilde{x}_t := \begin{cases} x_t, & t = 0,1,...,T-1 \\ 0, & T \le t \le L-1 \end{cases} ,$$

i.e. if one expands the orignal signal by adding L-T zero's, then the DFT of $\tilde{x}_t$ is given by

$$\tilde{X}_m := \sum_{k=0}^{L-1} \tilde{x}_k e^{-i(2\pi/L)km}, \quad m = 0,1,..., L-1 ,$$

i.e. $\tilde{X}_m$ is defined for the frequencies $\lambda_m = (2\pi/L)m$, m = 0,1,..., L-1. Since, however, $\tilde{x}_t = 0$ for $t \geq T$, it holds that

$$\tilde{X}_m := \sum_{k=0}^{T-1} \tilde{x}_k e^{-i(2\pi/L)km}, \quad m = 0,1,...,L-1 .$$

This technique of expanding a finite signal by addition of "zero's" allows a finer sampling of the frequency domain in calculating the DFT.

Since the entire frequency information of a signal is contained in its DFT, it appears appropriate to postulate a desired amplitude function *directly* and then to multiply the DFT by it. The filtered series is then obtained by the inverse transform IDFT. If a series is to be filtered by a low pass filter for instance, then one multiplies the DFT-values in the pass band by one and by zero in the stop band. A transition band must be implemented, however, between the pass- and stop band. Otherwise, high frequency oscillations emerge in the filter ouput (Gibb's Phenomenon). Such oscillations are due to the non-continuity of the amplitude function, as may be found by an ideal rectangular low pass amplitude. It is therefore advisable - also with respect to another reason to be presented shortly - to provide for a "very smooth" transition band. Since even very short transition bands suffice in order to avoid Gibbs's Phenomenon, it is possible to derive practically ideal filters in this manner. This holds naturally also for other types of filters such as, for example, high- or band pass filters or filters with non-standard amplitude characteristics. However, it is of important practical significance that the phase functions during the filtration in the frequency domain always assume the value zero over the entire frequency domain. Exact amplitude functions and zero phases may thus be simultaneously realized.

Unfortunately, however, a filter designed and implemented in the above prescribed manner exhibits a pattern at both ends of the series which is not accessible to interpretation. This *boundary problem* may be explained by the *finiteness* of practical time series. By comparison of two filter outputs corresponding to the results obtained for a filter of infinite length and a filter of finite length, one can define a "filter-error". The development of this filter error over time may then be clearly interpreted.

Between the filter input and the filter output the following relationship exists for both signals:

$$y_t = \sum_{k=-\infty}^{\infty} h_k x_{t-k} \quad \text{and}$$

$$\hat{y}_t = \sum_{k=-\infty}^{\infty} h_k I_{[1,T]}(t-k) x_{t-k} ,$$

whereby $I_{[1,T]}(t)$ represents a *data window* with the following property,

$$I_{[1,T]}(t) = \begin{cases} 1 & \text{for } 1 < t < T \\ 0 & \text{otherwise} . \end{cases}$$

$\hat{y}_t$ is the filter output for the *finite* signal $x_t, t = 1,...,T$, which represents a segment out of a signal of infinite length $x_t,...,1,0,1,....$ $\hat{y}_t$ is the "true" filter output, since it corresponds to an input signal of *infinite* length. A comparison of y_t and $\hat{y}$ for t = 1,...,T is illustrative of the sensitiveness of the signal upon the filter results. Taking the difference of both outputs, one obtains

$$y_t - \hat{y}_t = \sum_{k=-\infty}^{t-T-1} h_k x_{t-k} + \sum_{k=t}^{\infty} h_k x_{t-k}$$

$$= \sum_k h_k x_{t-k}, \quad \begin{cases} k \leq t-T-1 \\ k \geq t . \end{cases}$$

Now defining a squared filter error as

$$FF_{t,T} := |y_t - \hat{y}_t|^2, \quad t = 1,...,T,$$

it holds that

$$FF_{t,T} := |y_t - \hat{y}_t|^2 = \sum_k |h_k|^2 \; |x_{t-k}|^2 .$$

Assuming that the Fourier-transform of the infinitely long signal x_t exists, we obtain

$$X(e^{i\lambda}) = \sum_{t=-\infty}^{\infty} x_t e^{-i\lambda t}$$

resp.

$$x_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{i\lambda}) e^{i\lambda t} d\lambda .$$

Thus, one obtains

$$FF_{t,T} \leq \sum_k |h_k|^2 \; |\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{i\lambda}) e^{i\lambda(t-k)} d\lambda|^2 .$$

Given

$$\left\| X(e^{i\lambda}) \right\|_{\infty} := \sup_{\lambda} X(e^{i\lambda})$$

it follows that

$$FF_{t,T} \leq \left\| X(e^{i\lambda}) \right\|_{\infty} \sum_{k} |h_k|^2 .$$

The time-dependent upper error limit is not only determined by the signal (via the Fourier transform resp. spectral density) but also by the filter itself. The more quickly $|h_k|$ converges to zero, the smaller is $FF_{t,T}$. The speed of convergency depends upon the form of the amplitude function of the filter. Amplitude functions with noncontinuities result in slower speeds of convergency as compared to amplitude functions that have a "smooth" pattern. Narrow transition bands affect the speed of convergence negatively. Narrow pass bands have the same effect. Considering the filter specific error allotment,

$$\delta_{t,T} = \sum_{k} |h_k|^2$$

$$= \sum_{k=-\infty}^{t-T-1} |h_k|^2 + \sum_{k=t}^{\infty} |h_k|^2$$

it follows for the causal IIR-filter that $h_k = 0$ given $k < 0$. Therefore, $\delta_{t,T}$ is minimal for $t = T$, i.e. for the series' right end. For non-causal filters - such as a zero phase filter - $\delta_{t,T}$ is maximal for $t = 1$ and $t = T$, i.e. on both ends $\delta_{t,T}$ exhibits a U-form pattern for such filters. This explains the observed behavior of the filter output for both boundaries (left and right hand side) of a series.

There are basically two possibilities to reduce the filter error $FF_{t,T}$. The first one is to use only such filters whose filter weights in the time domain h_k fade very quickly. This implies, however, that one forgoes having exact amplitude functions. If this should not be the case, then one is only left with the possibility of attempting to keep the supremum of the Fourier-spectrum $||X(e^{i\lambda}||_{\infty}$ as small as possible. Naturally, in so doing the given signal x_t may only be manipulated in such a manner that is neutral with respect to the definitive filter output. This may occur in a relatively simple way. For example, if a low pass filter is to be applied to a signal, then - after specification of the amplitude function - the respective Fourier-spectrum of x_t, $t = 1,...,T$ must be estimated and the frequency determined at the supremum of the spectrum. If this frequency is located in the pass band, then a corresponding specified harmonic component of this frequency must be eliminated. The modified signal - whose Fourier-spectrum now has a smaller supremum as the orignal signal and therefore also a smaller filter error - is then filtered in the above described manner. The previously eliminated component is then added again to the filter output. These steps are of course redundant if the corresponding frequency of the supremum is located in the stop band. This procedure which can be described here only rudimentarily and simplified is usually applied *iteratively*. In so doing, the filter error is reduced iteratively, although it is not entirely eliminated. In general, filter outputs at the boundary may also be interpreted.

7.3 Global und Local Filters

The low pass filters considered above may be characterized as *global* filters. Global in the sense that they process the frequency information of the *total* series to be filtered. Unfortunately, global filters have a property which may appear to be disturbing under certain circumstances. This property reflects the fact that given new observations, previously filtered values may also change not only for the most recent observations. This is because the frequency information of a time series may change substantially by updating.

Disregarding for the moment the boundary, as a rule past changes are usually quite small. Yet for some applications even small changes may lead to problems of interpretation. Under such circumstances, it is advisable to use a *local* filter.

A *local* low pass filter may be obtained by not using the entire time series as filter input. Rather, filtering is carried out only for a *sliding span* of uneven length, using the same amplitude function as for the global low pass filter. The consequence of this is that only the respective (time-dependent) *local frequency information* is processed by the filter. The corresponding *filtered value* is the one which is aligned with the *center of the sliding span*. Studies have shown that a span of length $L = 13$ may be employed. At the boundary such a local filter only exhibits *six* observations which may change by updating. All other observations are *absolutely stable*.

Practical examples of local low pass filters are presented in Chapter IV. In particular, this chapter presents also comparisons to global filters as well as to smooth components due to the CENSUS procedure.

PC-compatible software (preferably for high end DOS-machines) exists for both global and local filters [39].

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34) A FIR-filter is a filter with a finite impulse response function. IIR-filter exhibit an infinite impulse response function. The impulse response function is that filter output which is generated when an impulse function is used as the filter input. The (discrete) impulse function is a function which for the time period t_0 takes on the value one and for all other time periods $t \neq t_0$ the value zero. See also further Chapter IV of this study.

35) The different versions of these procedures are not separately discussed here since they offer no special new aspects of interest.

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III. Properties of the CENSUS-X-11 Procedure in the Frequency Domain

1. Introduction

Seasonal adjustment procedures may be evaluated and compared according to a variety of quite different aspects. The following discussion limits itself to two properties of the CENSUS-X-11 procedure that have a large practical relevance. The first aspect deals with the *phase properties* of this seasonal adjustment procedure, whereby the second aspect has to do with the *amplitude properties*.

The first aspect stated above examines the question of whether phase shifts between the original time series and the adjusted series occur and if so to what extent (measured in time units). The term *adjusted* may be used to imply both the seasonal adjusted time series as well as the *smooth component*. This is especially important for *current and recent* data points of a time series, since in practice these adjusted values are commonly used for system diagnostic purposes. For instance, if a specific procedure exhibits a considerable phase shift at the actual end, this must be taken into consideration by the diagnosis. The adjusted values obtained must then be *redated* with regard to the corresponding time units (usually months or quarters). Furthermore, it must be considered whether or not they are still adequate at all to be used as the basis of diagnosis. Of course the ideal situation is one of a zero phase or quasi-zero-phase, i.e. given a phase that is smaller than the underlying time unit (month, quarter, etc.).

As the following discussion will make clear, the procedural phase properties should, however, not be considered independently of the procedural amplitude properties (and vice versa). The reason for this is not only the fact that direct relationships exist between the phase and amplitude of a procedure, but also because an isolated consideration of only one of the above mentioned aspects can lead to misinterpretations with respect to the *"quality"* of the procedure.

Knowledge of the amplitude characteristics of a given procedure also permits inferences about the underlying component definitions, which are however, often not explicitly or unprecisely stated. In particular, with the aid of the amplitude characteristics the question may be analysed whether or not a procedure defines a specific component in the same manner throughout the series. For example, does the procedure imply the same component definition in the middle of the series, as well as at the end of the series? If this is not the case, then strictly speaking the respective adjusted values are actually not comparable, since they are founded upon different definitions.

2. Theoretical Aspects of Determining the Amplitude and Phase Functions of the CENSUS-X-11-Procedure

In practice, an analytical determination of the phase- and amplitude characteristics of most of the commonly used seasonal adjustment procedures, especially with regard to the CENSUS-X-11 procedure, is usually not possible. The reasons for this are manifold. For instance, some procedures implement non-linear filters (if not throughout the entire procedure, then at least in many important steps of the procedure), or filters may be used iteratively, thereby leading to a total filter which may possess properties that are very difficult to grasp analytically if they even may be determined at all. Of particular interest, however, is the fact that most procedures use for adjustment of the most recent values different procedures than analysing the central values of the series. Basically, this implies that time-variant filters are the most commonly used ones. The analysis of the most recent values is especially difficult. All the above stated problems also emerge when taking the CENSUS-X-11 procedure into consideration.

Due to the reasons mentioned above, in the following we choose an underlying empirical-experimental approach in determining the amplitude- and phase characteristics of the CENSUS-X-11 procedure founded upon appropriate test-signals. Let it be noted here, that the application of empirical studies based upon concrete time series which are in principle possible, were purposely not undertaken here to avoid any objection that the results were dependent upon the specific time series examined.

The following considerations demonstrate how appropriate test signals may be derived[1)].

If X_t represents the desired test- (or input-) signal and Y_t the respective output signal of the seasonal adjustment procedure, then the relationship between both these signals may be expressed as follows:

$$F : X_t \Rightarrow Y_t , \quad t = 1,...,T .$$

F denotes a filter that transforms the input signal into the output signal, i.e. the adjustment procedure in its entirety is quasi interpreted as a single filter.

Given the end point of a series, the above description must be modified such that the filter is usually time-variant. (This holds naturally also for the beginning of a series. However, these are usually of less practical importance since such adjusted values are at most of interest under a retrospective analysis objective.) Thus,

$$F_t : = X_t \Rightarrow Y_t ,$$

whereby t now denotes time points in the *boundary domain*. The starting point of this domain is in general different for various procedures. Frequently, it may only be ascertained empirically.

Now using the test signal

$$X_t = \sin(\lambda t + \delta)$$

and the filter F resp. F_t, then one obtains the following filter output

$$F_t : X_t \Rightarrow Y_t = A_t(\lambda)\sin[\lambda t + \delta + \Phi_t(\lambda)] .$$

$\Phi_t(\lambda)$ denotes the phase shift between the input and output, that is caused by the filter F resp. F_t, i.e. due to the adjustment procedure. The phase function $\Phi_t(\lambda)$ is - as mentioned above - especially of interest at the boundary. In addition, it should be mentioned here that the phase function at the center of series is practically equal to zero for most procedures, since in contrast to the boundary symmetric filters may be applied there.

$A_t(\lambda)$ represents the amplitude function of the filter F resp. F_t. It shows to what extent the corresponding "*power*" of different frequencies resp. frequency bands of the procedure are eliminated, attenuated, or even amplified. A *good* seasonal adjustment procedure should only eliminate the *seasonal "power"*, but not affect the other frequency bands.

As may be seen from the above stated input-output relationship, the empirical analysis presented here is founded upon the theory of linear filters. This implies that the results obtained for procedures that use non-linear filters must only be understood as approximations.

Since both $A_t(\lambda)$ and $\Phi_t(\lambda)$ are to be determined for all time points t = 1,2,...,T and all frequencies λ, two filter outputs Y_t^1 and Y_t^2 are necessary. They are obtained by an appropriate choice of the parameter δ.

Given $\delta = \delta_1$ and $\delta = \delta_2$, the following outputs are obtained

$$\begin{aligned} Y_t^{1,2} &= A_t(\lambda)\sin[\lambda t + \Phi_t(\lambda) + \delta^{1,2}] \\ &= A_t(\lambda)\{\sin[\lambda t + \Phi_t(\lambda)]\cos\delta^{1,2} + \cos[\lambda t + \Phi_t(\lambda)]\sin\delta^{1,2}\} \\ &= \begin{cases} A_t(\lambda)\sin[\lambda t + \Phi_t(\lambda)] , & \delta^1 = 0 \\ A_t(\lambda)\cos[\lambda t + \Phi_t(\lambda)] , & \delta^2 = \pi/2 , \end{cases} \end{aligned}$$

and therefore

$$\begin{aligned} Y_t^1 &= A_t(\lambda)\sin[\lambda t + \Phi_t(\lambda)] \\ Y_t^2 &= A_t(\lambda)\cos[\lambda t + \Phi_t(\lambda)] , \end{aligned}$$

i.e. the respective test signals are

$$\begin{aligned} Y_t^1 &= \sin\lambda t \\ Y_t^2 &= \cos\lambda t . \end{aligned}$$

From the above equations the empirical amplitude - resp. phase functions may be derived. Consequently,

$$A_t(\lambda) = \sqrt{[Y_t^1]^2 + [Y_t^2]^2} .$$

Now dividing both filter outputs Y_t^1 and Y_t^2, one obtains

$$\frac{Y_t^1}{Y_t^2} = \tan[\lambda t + \Phi_t(\lambda)] .$$

Therefore, the empirical phase function is

$$\Phi_t(\lambda) = \arctan \frac{Y_t^1}{Y_t^2} - \lambda t .$$

In practice, however, the usual arctan-function is not suitable to determine the phase function since it is only defined for the interval $[-\pi/2 , \pi/2]$. Hence, the ATAN2-function which is defined in the interval $[-\pi, \pi]$ is additionally applied.

To enhance the interpretation of the derived phase functions it is advisable to transform them into the time domain, since phases expressed in radians are not necessarily very informative.

3. Experimental Design

In order to determine the amplitude and phase of the CENSUS-X-11 procedure empirically 60 test signals $X_t^1 = \sin\lambda t$ and $X_t^2 = \cos\lambda t$ of length T = 200 have been generated respectively, whereby the frequency interval $[0 , \pi]$ is sampled at the frequencies $\pi k/60$, k = 1,2,...,60. Amplitude and phase have been determined for the series center as well as for the boundary. As center the time points t = 100 to t = 170 and as boundary the time points t = 171 to t = 200 have been chosen.

This was done under the supposition that monthly series are the most commonly adjusted. An analogous study of e.g. quarterly data could also naturally be carried out.

The amplitude function of an *ideal* seasonal adjustment procedure should demonstrate the pattern of an *ideal* multiple notch filter, since such a function guarantees that only the "*power*" of seasonal frequencies are filtered out. Such an amplitude filter may be realized e.g. with a recursive filter [2], as demonstrated below in Figure 70. It differs only marginally from the ideal amplitude function.

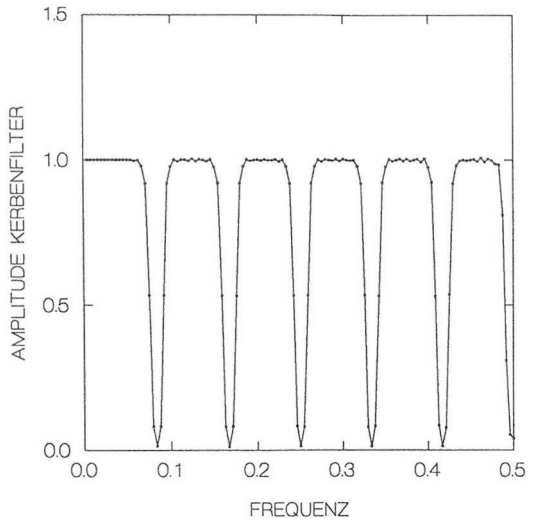


Figure 70

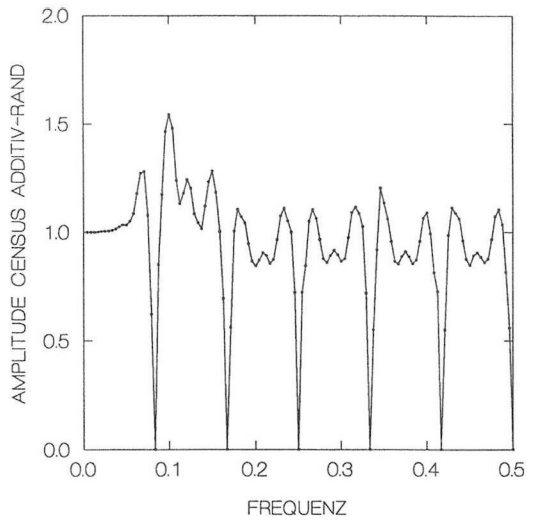


Figure 72

As common for most procedures in general, in addition to the seasonal adjusted values the CENSUS-X-11 procedure also estimates a *smooth component*. The smooth component should contain no seasonal "*power*" or *noise*. It should ideally be the result of an exact low pass filter. The amplitude function of such a filter exhibits the value one in the frequency domain [0, 1/12) and zero otherwise. This should serve as a *standard of reference* for the smooth component.

4. Results for the Amplitude Functions

The obtained results are presented graphically. Figures 71-74 illustrate the empirically derived amplitude functions of the CENSUS-X-11 procedure for the series center and boundary of the *seasonal adjustment* both for the *additive* as well as for the *multiplicative* version.

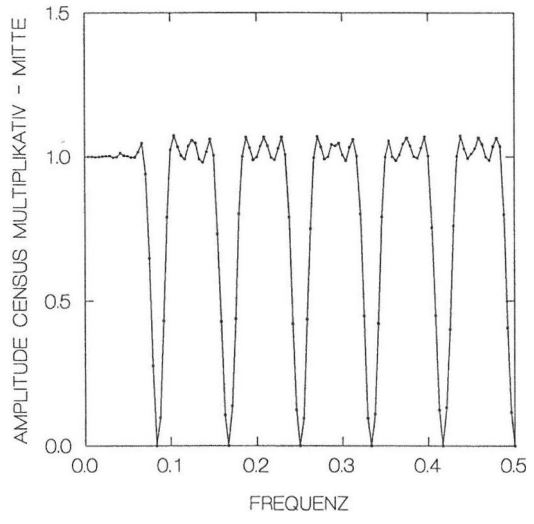


Figure 73

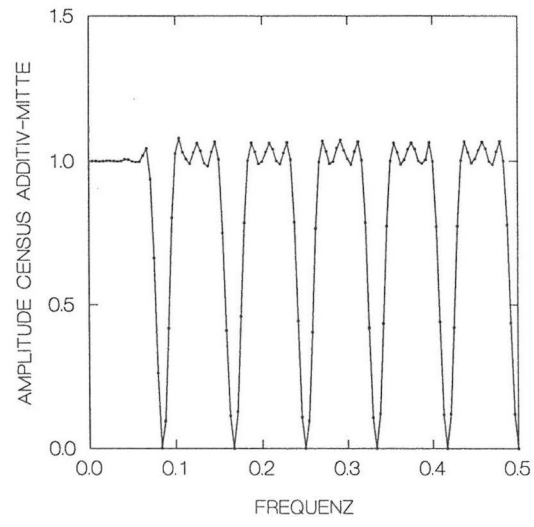


Figure 71

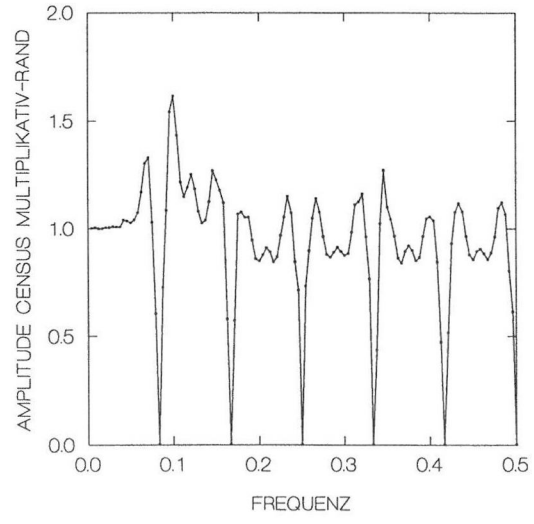


Figure 74

Figures 75 and 76 illustrates jointly the amplitudes for both the center and the boundary.

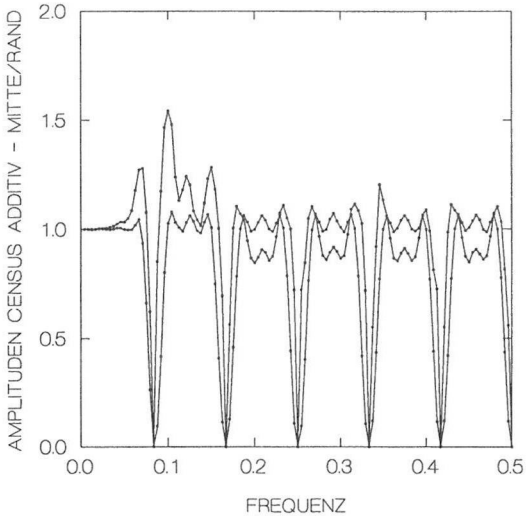


Figure 75

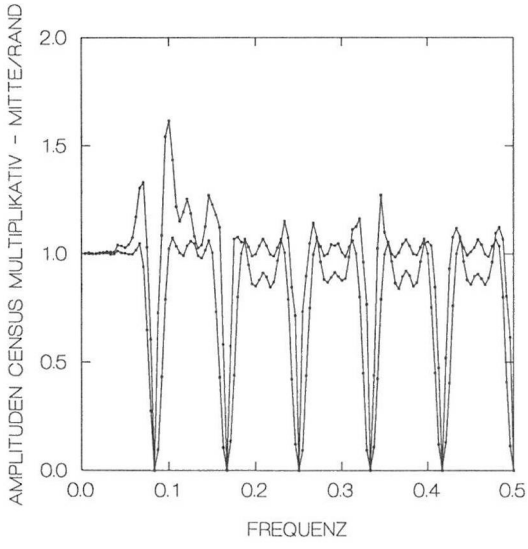


Figure 76

For the series center and boundary, figures 77-80 exhibit the amplitude functions of the *smooth* component for the additive resp. multiplicative version of the CENSUS-X-11 procedure.

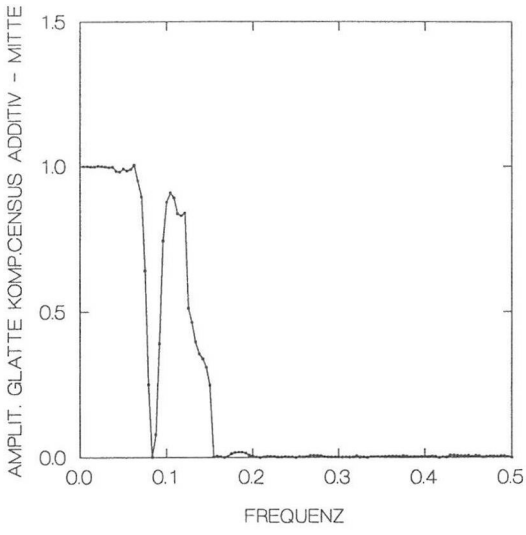


Figure 77

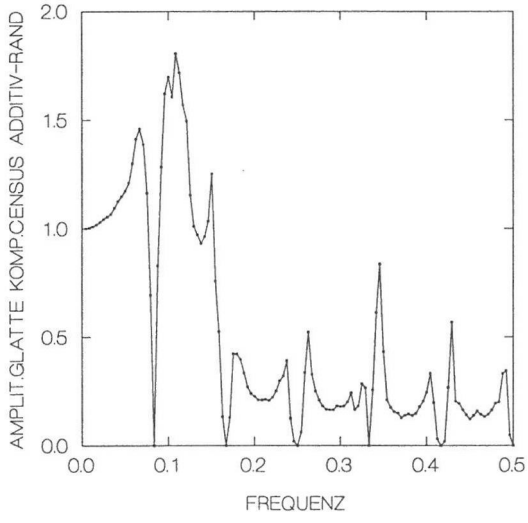


Figure 78

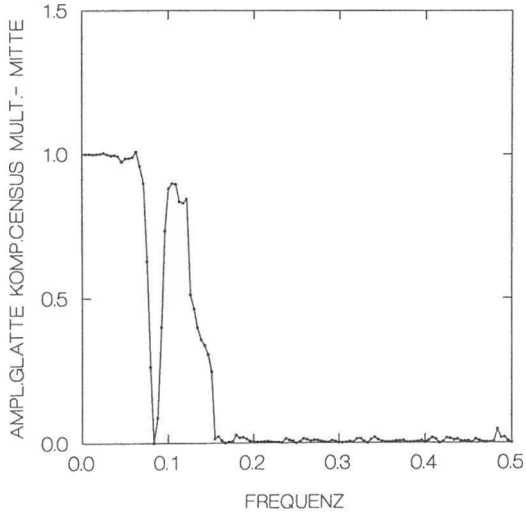


Figure 79

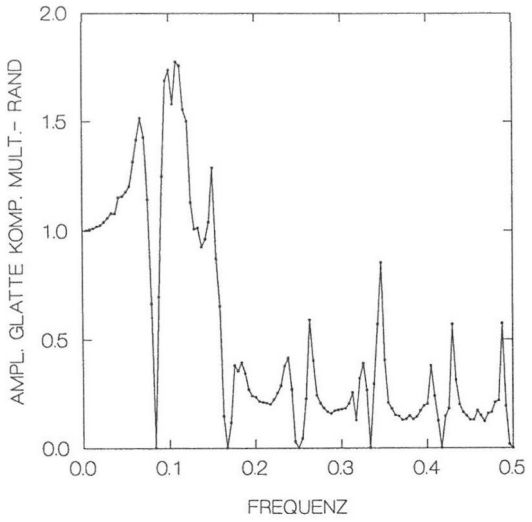


Figure 80

In figures 81 and 82 the amplitude functions for the series center and boundary are presented jointly.

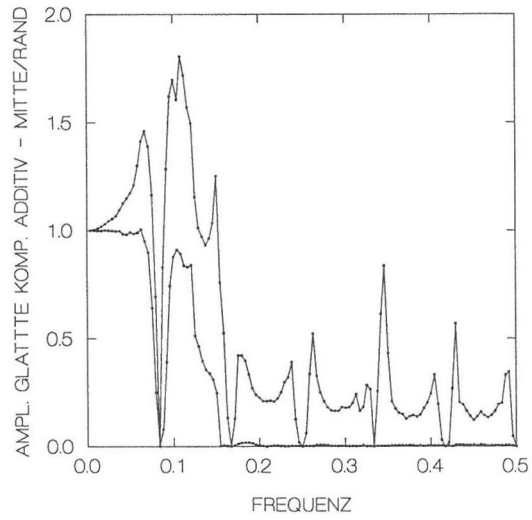


Figure 81

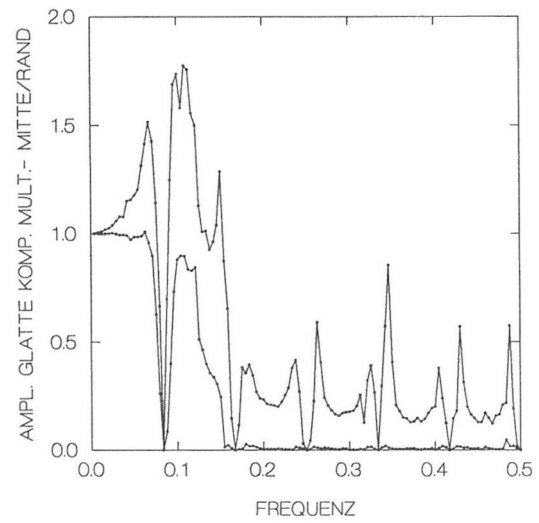


Figure 82

5. Results for the Phase Functions

The results derived for the phase functions are presented analogously to those derived for the amplitude functions, i.e. both the *additive* as well as the *multiplicative* version are presented, whereby we differentiate additionally between phases in *radian* and phases in *time units*. Figures 83-86 illustrate the phases of *seasonal adjustment* for the *additive* version. Figures 87 and 88 picture jointly the series center and boundary.

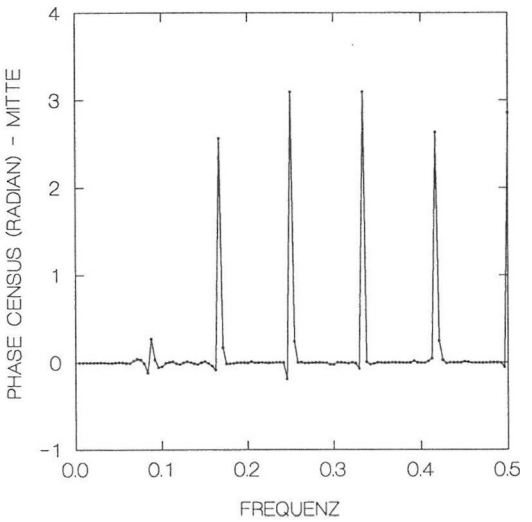


Figure 83

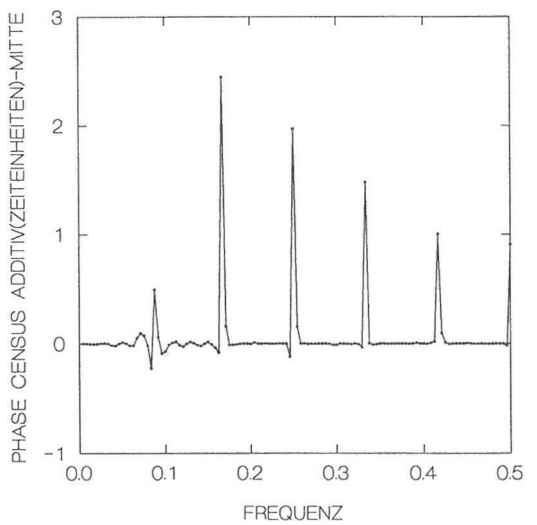


Figure 84

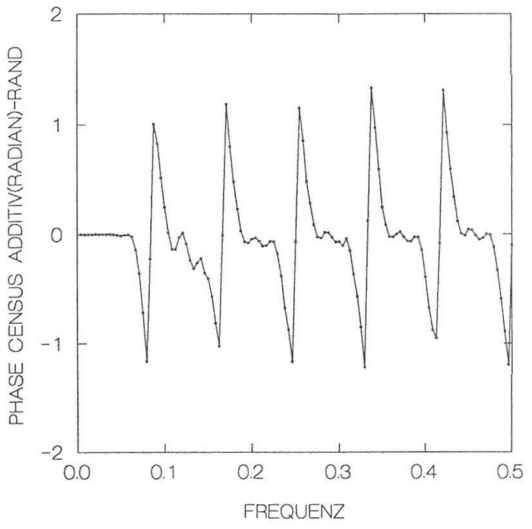


Figure 85

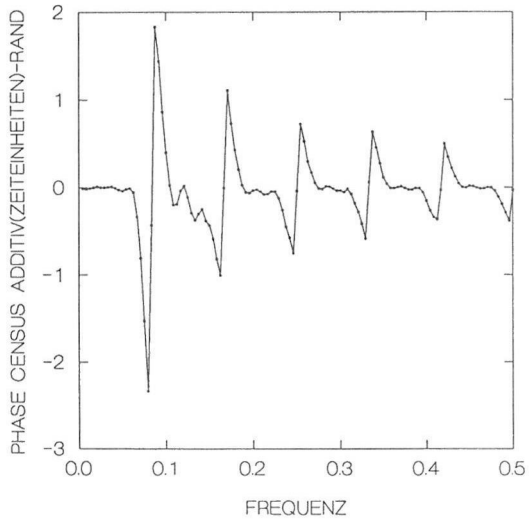


Figure 86

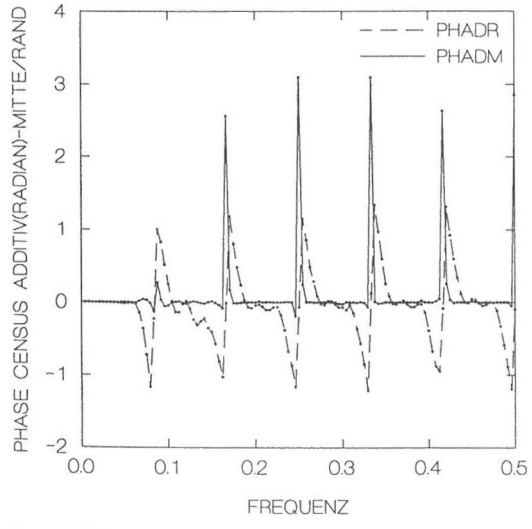


Figure 87

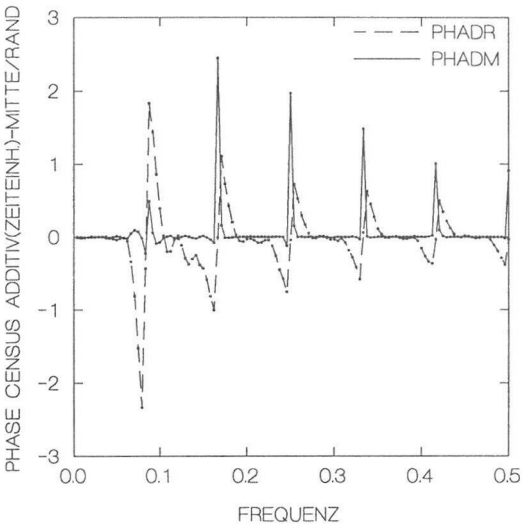


Figure 88

Figures 89-92 exhibit the phase functions of the *multiplicative* version and Figures 93 and 94 present the center and the boundary jointly.

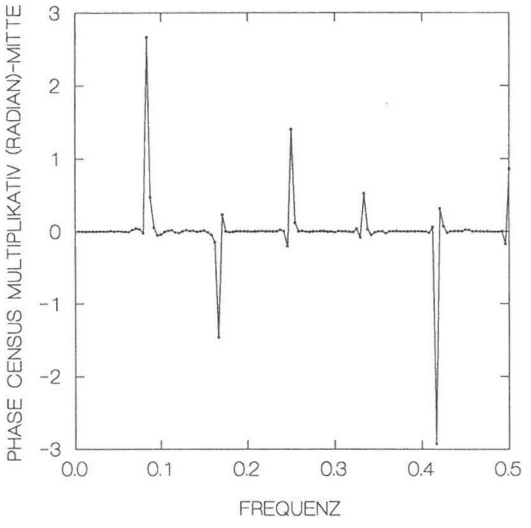


Figure 89

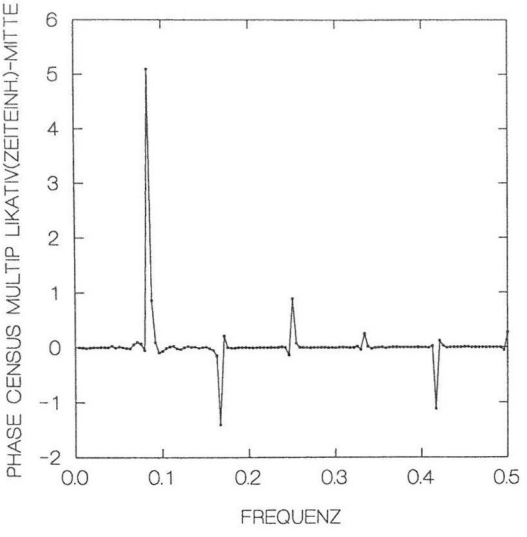


Figure 90

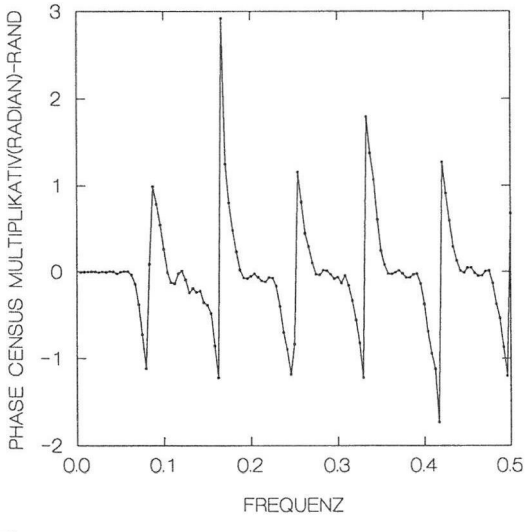


Figure 91

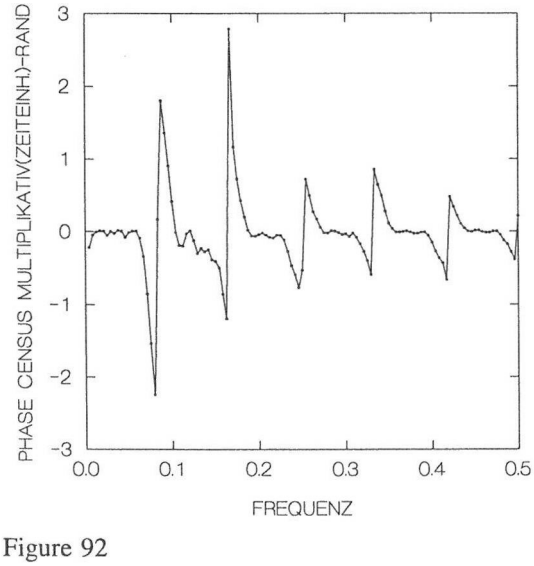


Figure 92

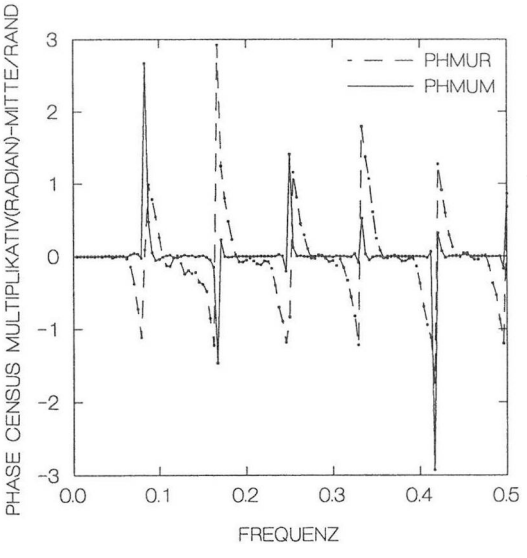


Figure 93

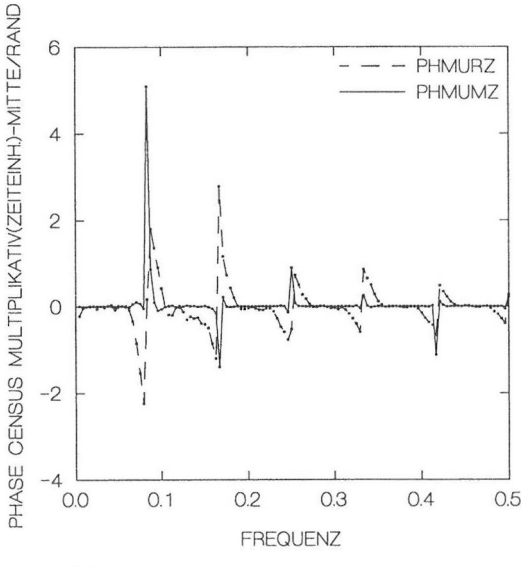


Figure 94

The phase functions of the *smooth component* of the *additive* version are given below in Figures 95-98. Figures 99 and 100, once again, demonstrate center and boundary jointly.

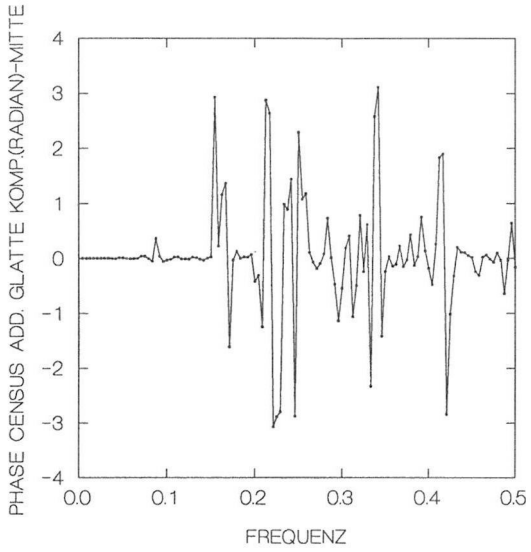


Figure 95

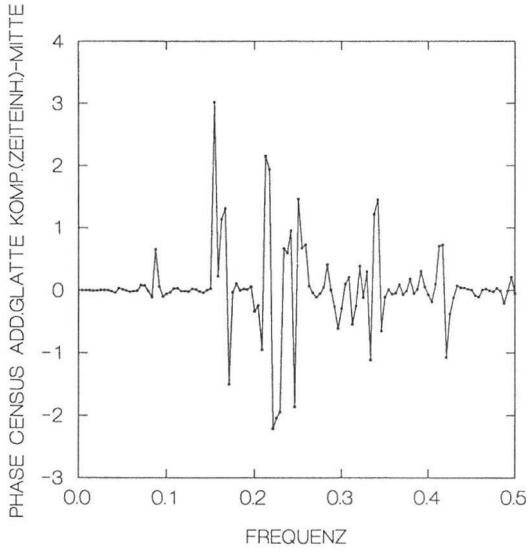


Figure 96

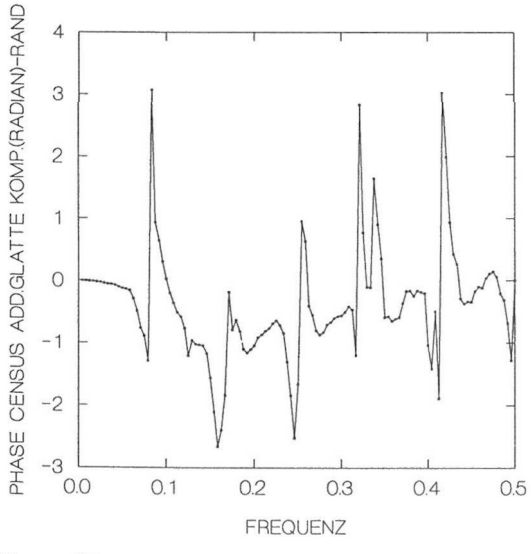


Figure 97

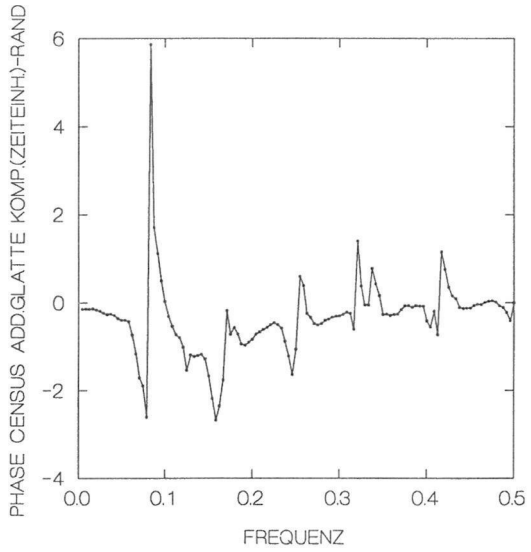


Figure 98

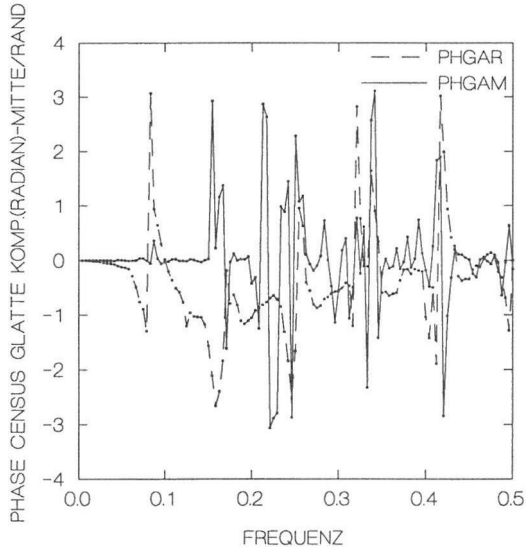


Figure 99

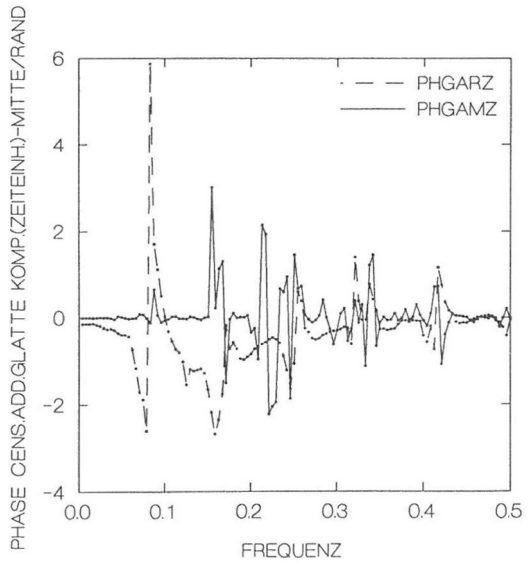


Figure 100

Finally, the phase functions of the *smooth* component of the *multiplicative* version are presented in figures 101-104 below. The combined graphing of center and boundary are given in Figures 105 and 106.

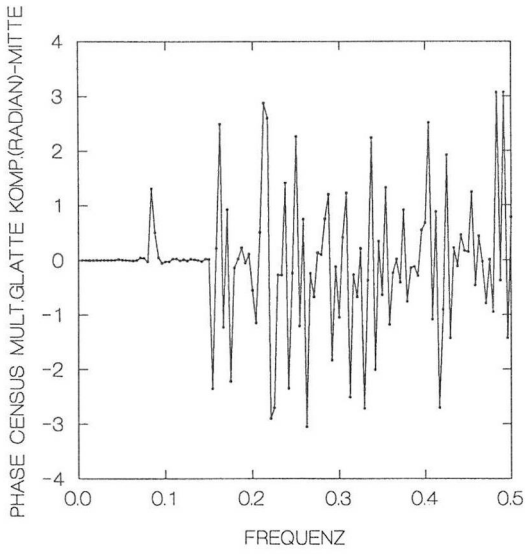


Figure 101

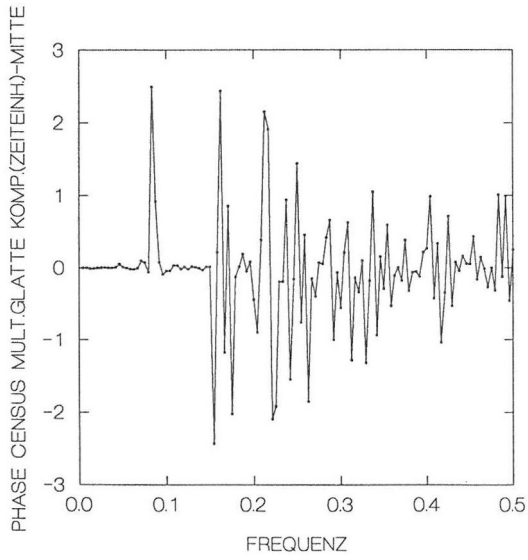


Figure 102

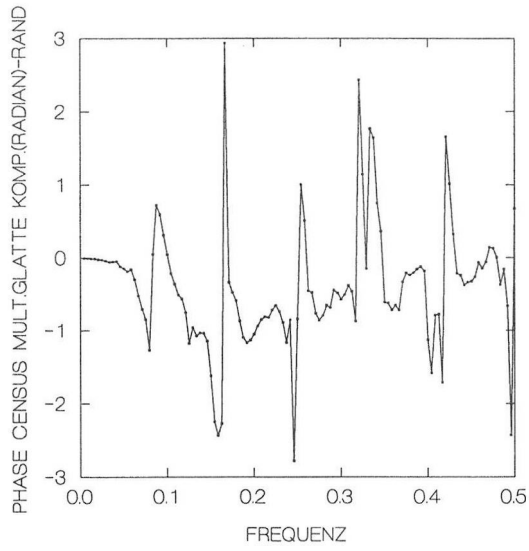


Figure 103

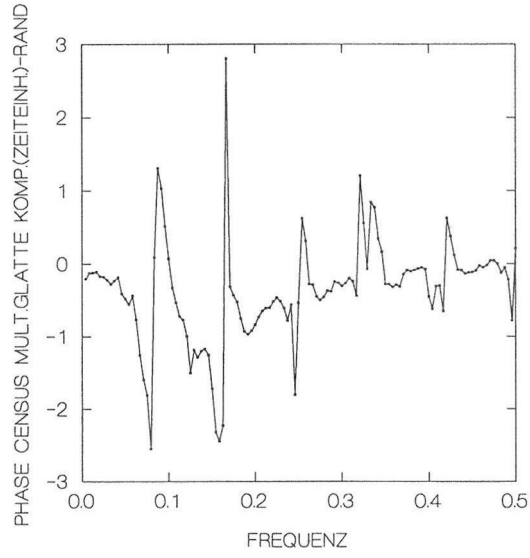


Figure 104

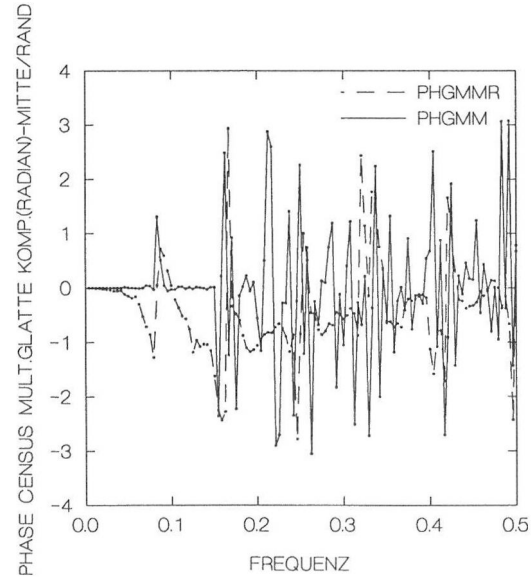


Figure 105

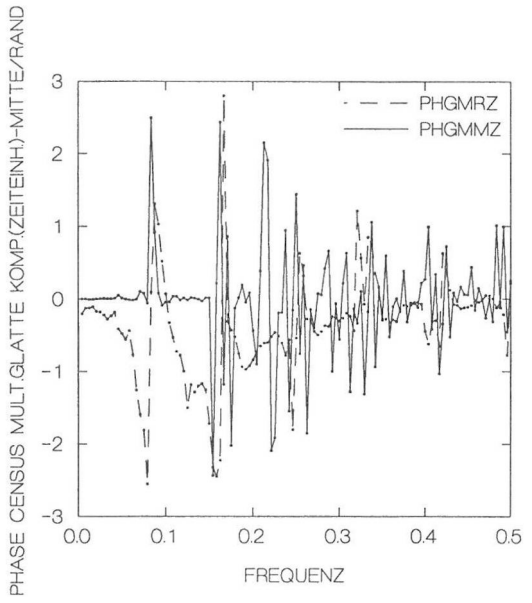


Figure 106

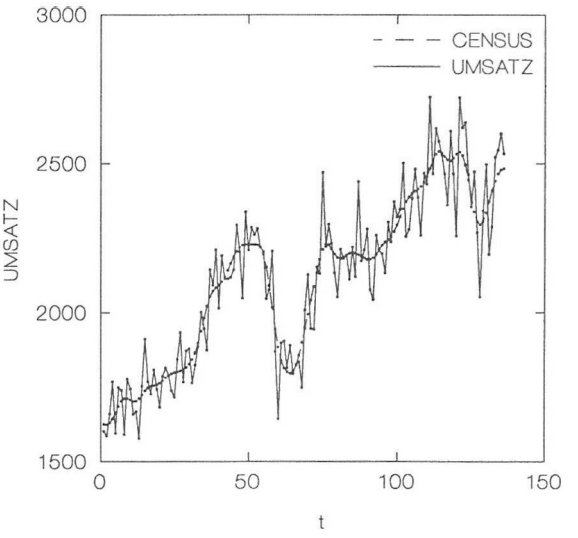


Figure 107

6. Interpretation of the Amplitude Functions

If one compares the *amplitude functions* of the *seasonal adjustment* of both versions of the CENSUS-X-11 procedure, basically no significant differences are to be observed. This holds for the series center as well as the boundary (cf. Figures 71 and 72 resp. Figures 73 and 74). Looking at the series center both versions exhibit a relatively good approximation of the amplitude of the notch filter as illustrated in Figure 70.

Both versions of the CENSUS-X-11 procedure display a *distorted amplitude* at the series end, whereby the degree of distortion in both cases may be classified as approximately similar (cf. Figures 72 and 74).

The amplitudes of the smooth components demonstrate that the CENSUS-X-11 procedure exhibits particularly *poor low pass filter properties*, even for the *series center*, which one would actually not expect. Examining Figures 77 and 79, both amplitudes display for the series center a significant *peak* between the first and second seasonal frequency - the amplitudes show a value of circa 0.9 - although they should be zero or at least approximately zero in this frequency domain. At the boundary of series this peak is further amplified (compare Figure 78 and 80).

The above fact is by no means of pure academic interest, since it has serious practical consequences. It is namely responsible for the observed oscillations of the smooth component. Thus, both versions of the CENSUS-X-11 procedure apparently exhibit a type of Slutzky Effect.

This Slutzky Effect may be demonstrated with real series. Figure 107 below is illustrative of a monthly series (company sales) together with its smooth CENSUS-X-11 (multiplicative) component.

That the smooth component exhibits a Slutzky Effect may be clearly seen by comparing it with the smooth component of an exact low pass filter (cf. Figure 108).

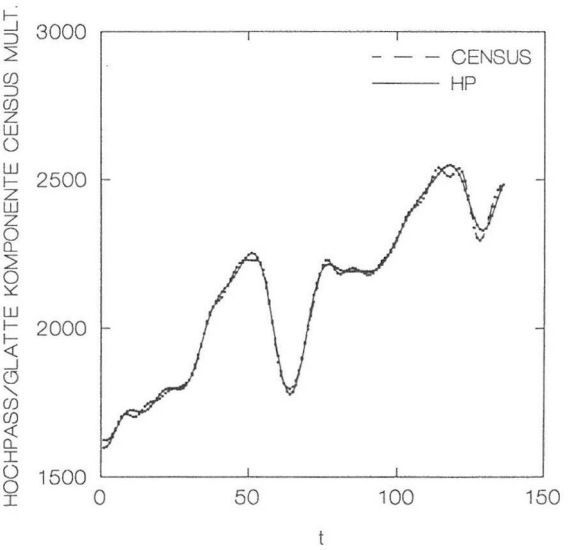


Figure 108

Obviously, the CENSUS-X-11 component oscillates about the low pass component with a period of circa 9 months, which approximately corresponds to the above mentioned *peak frequency*. Optically, these oscillations may be observed quite lucidly by examining a segment of Figure 108 as presented in Figure 109 below.

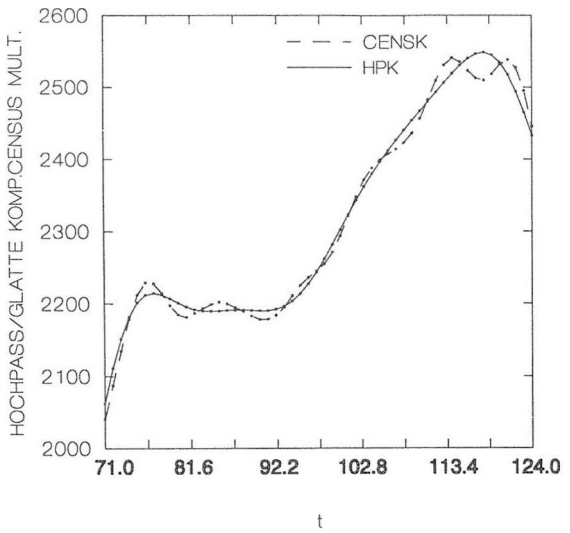


Figure 109

Of course, appreciation of the fact that the smooth component of the CENSUS-X-11 procedure contains a Slutzky Effect is not entirely new. Twenty years ago SCHÄFFER demonstrated by using relatively comprehensive and complicated time series models that the smooth CENSUS-X-11 component may exhibit a Slutzky Effect[3]. Unfortunately, his results failed to receive their due attention.

Looking at the smooth component at the boundary, the resulting CENSUS-X-11 amplitudes are even poorer than those obtained for the center (cf. Figures 78 and 80). They appear to have apparently nothing more in common with the amplitude of a low pass filter. In addition to the fact that the observed *peak* is approximately doubled in height compared to the center, it is further evident that the high frequency content of a series is not adequately filtered out (after the first seasonal frequency the amplitudes should be zero or almost zero). This leads to *highly volatile* boundary values of the smooth component, a phenomenon that has been well known for years to intensive user of the CENSUS-X-11 procedure (e.g. the "DEUTSCHEN BUNDESBANK"), for which, however, up to date apparently a theoretical explanation was lacking.

7. Interpretation of the Phase Functions

For both versions of the CENSUS-X-11 procedure the phases in the *series center* (either in radians or in time units) are *practically equal to zero* in the first (and decisive) frequency band, which extends to the first seasonal frequency (consider Figures 83 and 84 as well as Figures 89 and 90). But this is not surprising, remembering that in the series center *symmetrical filters* are exclusively applied.

Of contrasting interest, however, is the phase behavior at the series end. There one observes basically the *same phase pattern as in the middle* of the series, although *asymmetric filters* are implemented (see Figures 85 and 86 resp. Figures 91 and 92).

Exactly the same situation is also to be found for the smooth component (cf. Figures 95 and 96 along with Figures 97 and 98). This finding is especially astonishing (particularly for the end), since the estimation of these components at the end utilizes strong *asymmetric filters.*

The unexpected good phase properties of the CENSUS-X-11 procedure at the boundary may be explained easily if one observes the amplitude and phase functions *together.*

Given an asymmetric filter there generally exists an *antinomy* between amplitude and phase, i.e. the more focused the amplitude is, the poorer (i.e. the larger) the phase is. Putting it somewhat differently, the favorable phase properties at the boundary of the CENSUS-X-11 procedure are purchased at the price of *destroying the amplitude function.* The surprising quasi-zero phases at the boundary therefore ultimately occur due to miserable amplitudes at the boundary.

In addition, let it be noted here that taking a methodological perspective on this point, the boundary values (the seasonal adjusted ones and even more so the values of the smooth component) are actually *not comparable* with their corresponding values in the series center. The reason for this is that the (realized) amplitude functions show the definition of the components in the frequency domain met by the procedures, or putting it more exactly, of the *operationalized* component defintions. Strongly divergent amplitude functions between series center and boundary as presented here are synonymous for *different* component definitions.

8. Estimation of the Smooth Component of Simulated Series with the CENSUS-X-11 Procedure

As presented above in section 6 of this chapter, the amplitude functions of the smooth component of the CENSUS-X-11 procedure must be characterized as especially problematical. In order to illustrate the difficulties involved in practical applications, first of all the smooth components of the four simulated time series discussed previously in chapter I, section 4 (see Figures 25, 29, 30 and 31) are estimated by CENSUS-X-11. These estimates are then contrasted with the smooth component obtained by a low pass filter.

Remembering the discussion from chapter I, section 4 above, these series are based upon the following simple model:

$$X_t = 2 + 0.9t + \sin(2\pi/12) + \varepsilon_t \quad , t = 1,2,...,100 \ .$$

The smooth component of all four series is thus defined by the linear function 2 + 0.9t. The only difference amongst the series is that the respective noise term ε_t exhibits a different variance. For series 1 (see Figure 25) it is equal to zero implying that the series has no noise, for series 2 (see

Figure 29) it equals 0.1, for series 3 (see Figure 30) it equals 1.0 and finally for series 4 (see Figure 31) it equals 2.0.

The following Figures 110-113 now present the corresponding estimated smooth components of these series using the CENSUS-X-11 procedure.

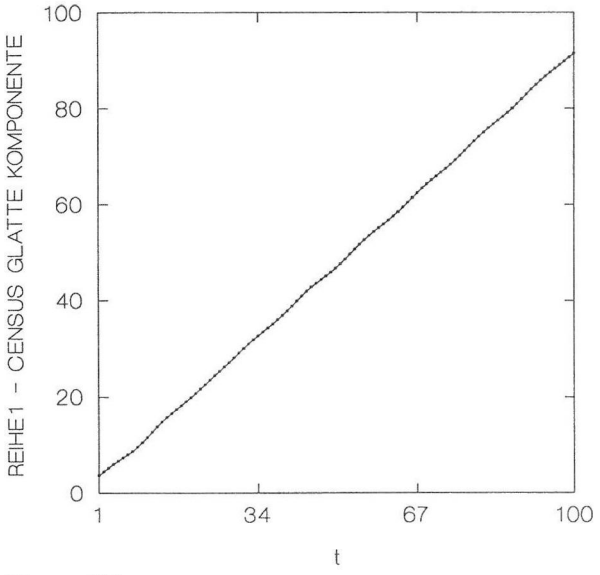


Figure 110

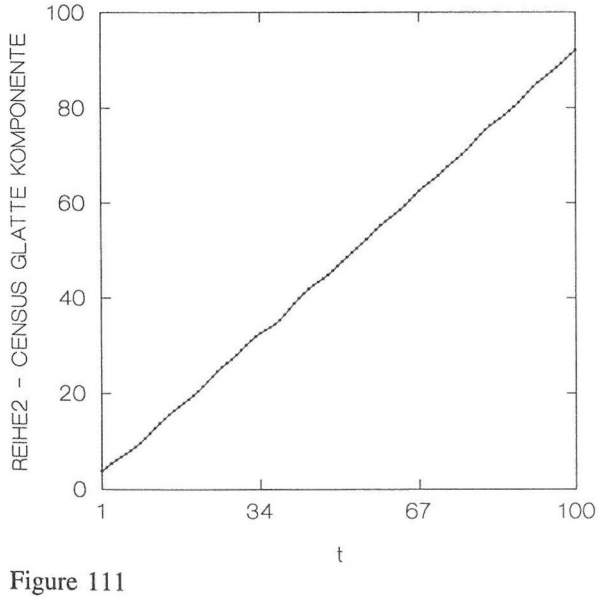


Figure 111

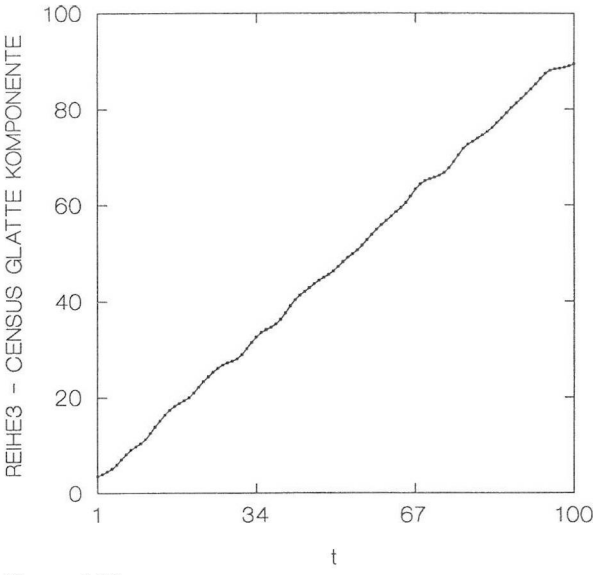


Figure 112

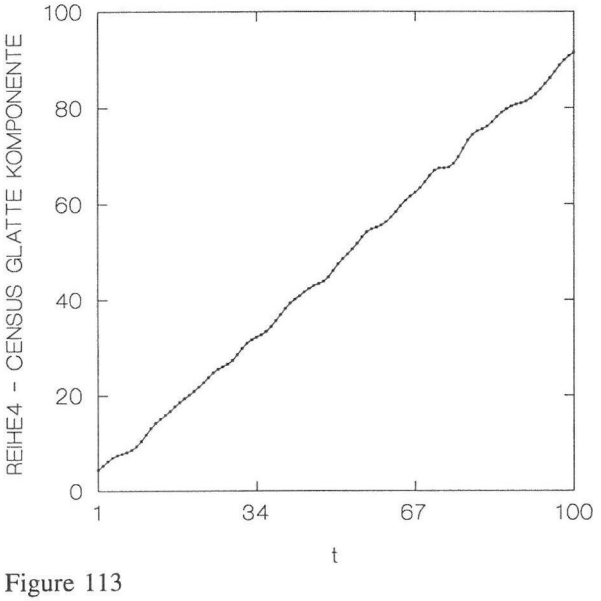


Figure 113

It is evident from the above figures that the smooth component of the CENSUS-X-11 procedure exhibits oscillations, which are especially striking for the series 3 and 4 (cf. Figures 112 and 113). However, they can also be seen in series 1 and 2, although not as evident.

For purposes of comparison, the filter output of a low pass filter for series 4 is presented in Figure 114 below.

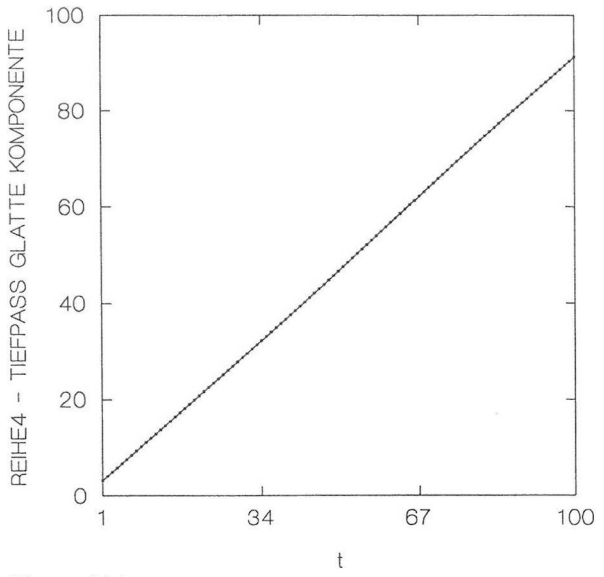


Figure 114

This smooth component can practically not be differentiated from a line, that is the adjusted series exhibits no oscillations. Similar results are obtained for the other three series so that an explicit presentation may be disregarded here.

The better results obtained throughout by applying a low pass filter may also be demonstrated with the aid of a *goodness-of-fit measure* by comparing the differences between the theoretical smooth component function 2 + 0.9t with the corresponding estimated function of the CENSUS-X-11 procedure as well as with the respective component of a low pass filter. For instance, if one uses the *mean squared error*

$$MQF_C = \sqrt{\frac{1}{100}\sum_{t=1}^{100}(GK_t - GKC_t)^2}$$

resp.

$$MQF_{TP} = \sqrt{\frac{1}{100}\sum_{t=1}^{100}(GK_t - GKTP_t)^2}$$

with

$$GK_t = 2 + 0.9t$$

GKC_t : Smooth component, CENSUS
$GKTP_t$: Smooth component, LowPass ,

then the following values are obtained:

	CENSUS-X-11	Low Pass Filter
Series 1 :	0.2275	0.0371
Series 2 :	0.2913	0.0965
Series 3 :	0.6333	0.1930
Series 4 :	0.7522	0.2983

In addition to this measure one can also, for example, calculate the standard deviations of the (percentage) relative differences, i.e. the standard deviations of the differences related to the theoretical values of the respective smooth components. Doing so for CENSUS-X-11 and for a low pass filter the following results emerge:

	CENSUS-X-11	Low Pass Filter
Series 1 :	3.26 %	0.55 %
Series 2 :	3.69 %	0.86 %
Series 3 :	2.85 %	2.38 %
Series 4 :	9.11 %	1.71 %

Looking at the above standard deviations one observes once again also that the values of the low pass filter are better.

Besides considering such or similar measures, more information may possibly be obtained by considering the differences themselves as time series. Related to this aspect is then an objective that may be formulated in the form of the following question: Do the differences still contain information that they "properly" should not contain?

The differences for the series adjusted by CENSUS-X-11 are presented below in Figures 115-118:

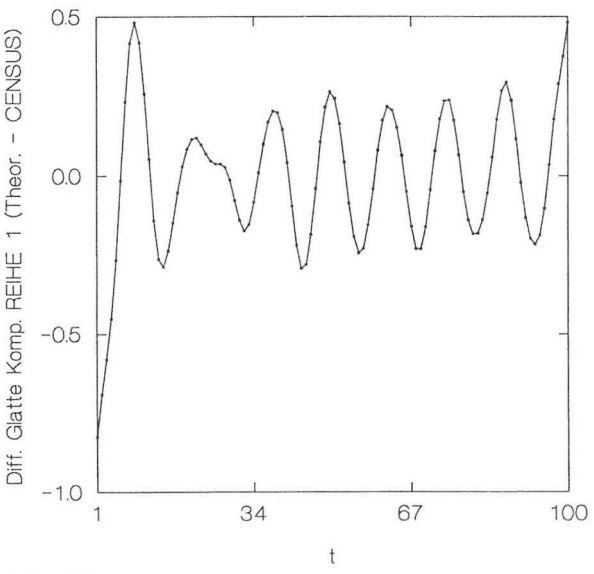


Abb. 115

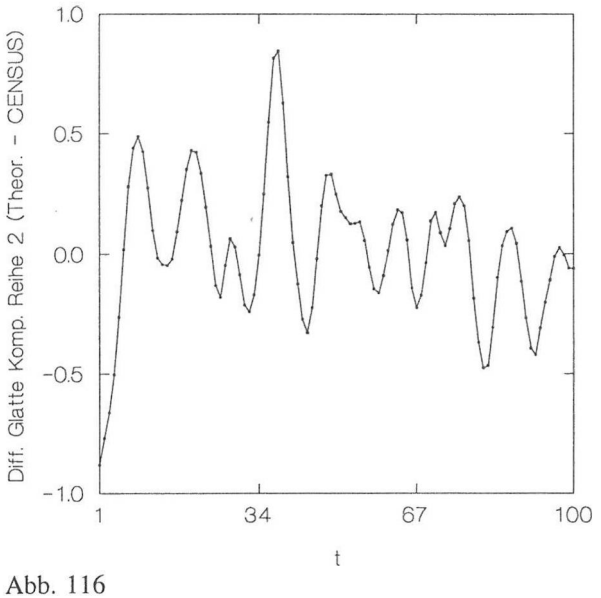


Abb. 116

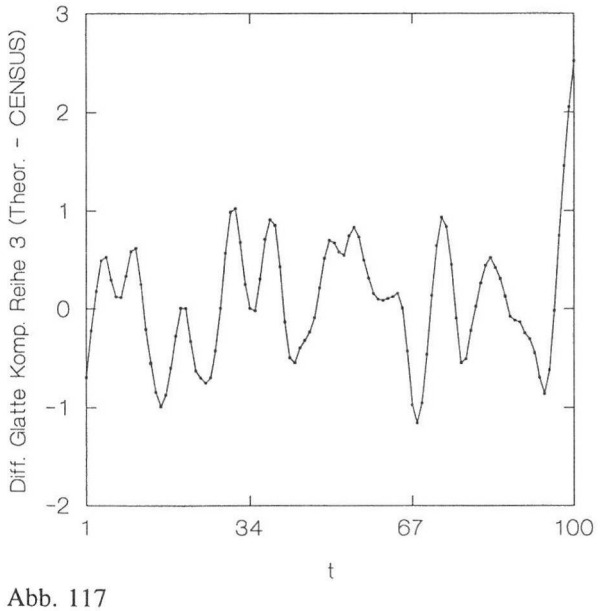


Abb. 117

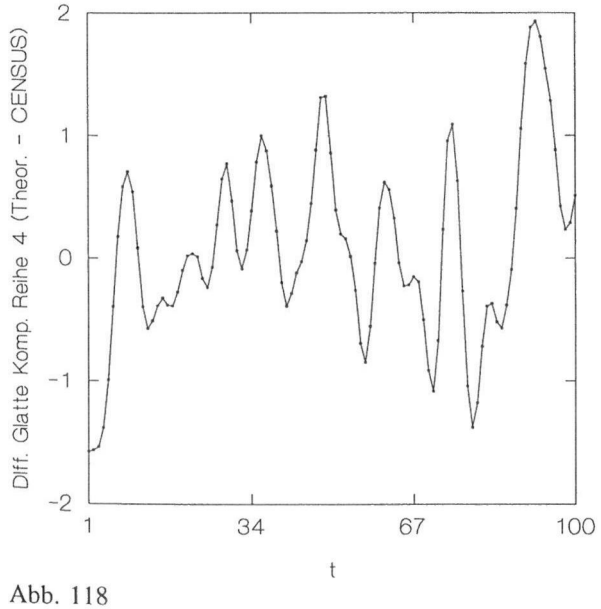


Abb. 118

It can be noticeably observed that all four difference-series exhibit rather regular short term oscillations. This may also be observed by looking at the estimated spectra of these differences, presented in Figures 119-122 below:

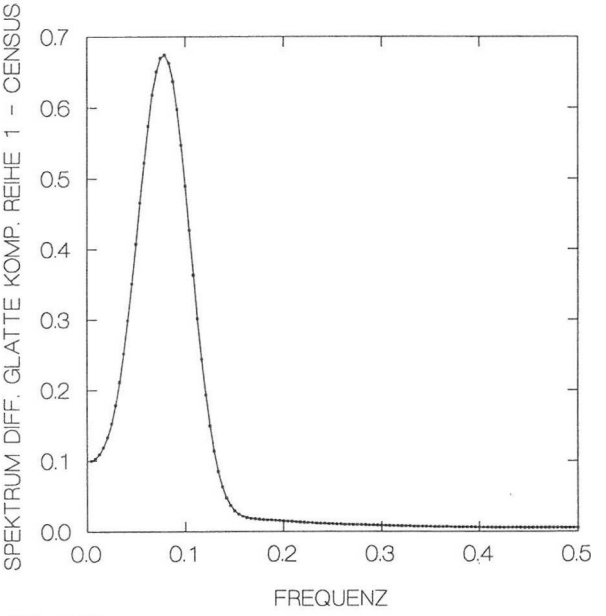


Abb. 119

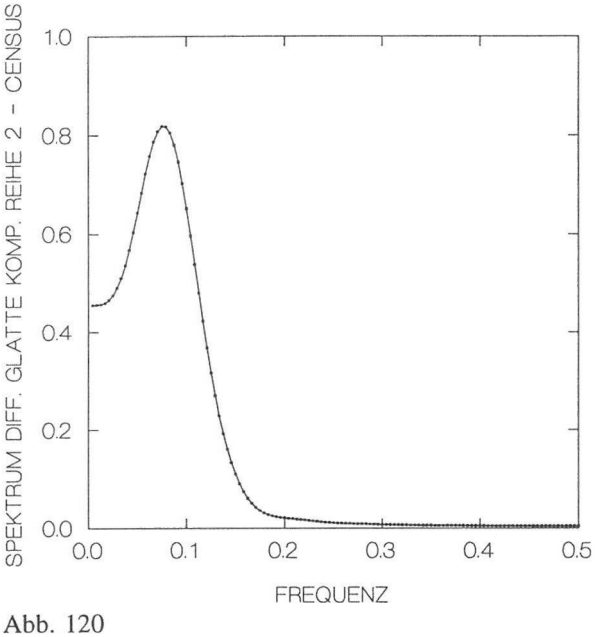


Abb. 120

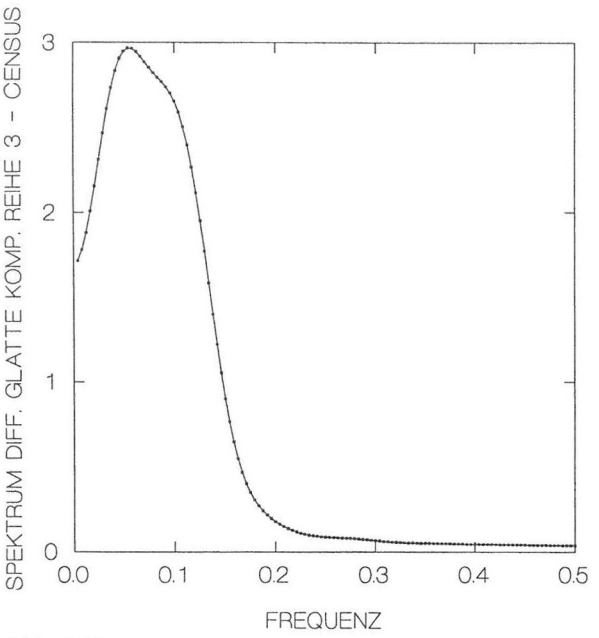


Abb. 121

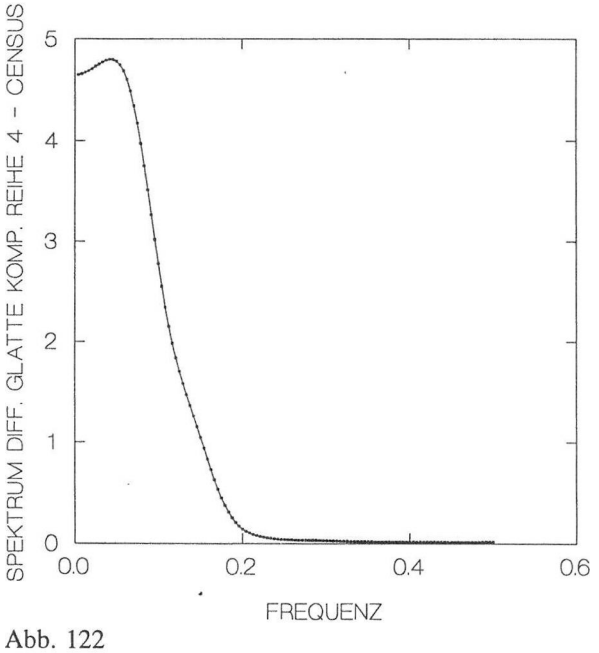


Abb. 122

For the first three difference series the corresponding *peak* in the spectrum is located at approximately 12, 13 and 18 months. The spectrum of the forth difference series does not permit a clear interpretation because the series exhibits a slight trend. If this trend is filtered out by a high pass filter, the spectrum results in the following pattern (see Figure 123):

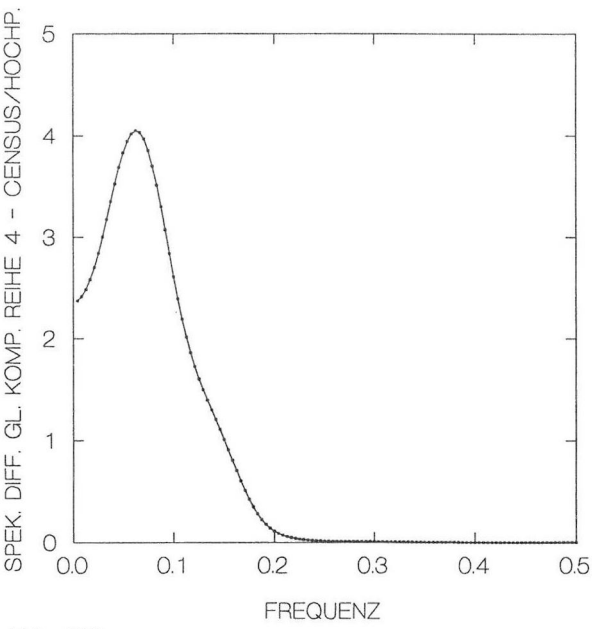


Abb. 123

The maximum of this spectrum is located at approximately 16 months.

Therefore these spectra confirm the previous optical impressions of accentuated, yet relatively short, cyclical movements of the above difference-series.

The difference-series of the low pass filter exhibit a similar pattern for all four series. Thus, they may be presented jointly in Figure 124.

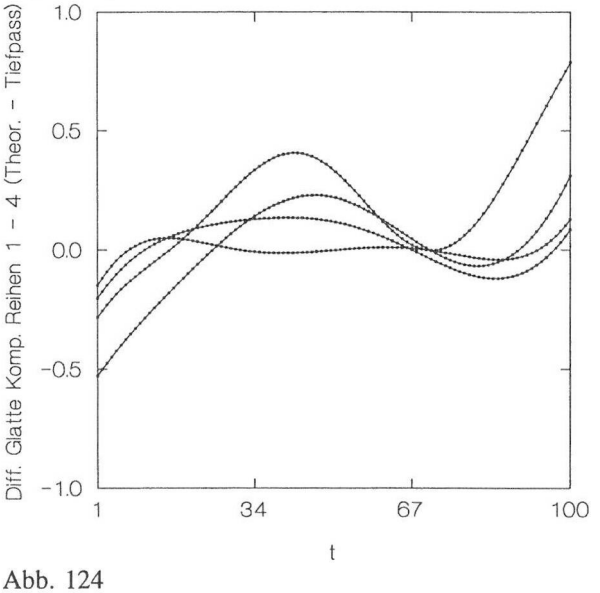


Abb. 124

Contrary to the CENSUS-X-11 differences, these differences exhibit a very smooth pattern without any recognizable cyclical movements. It is to be expected therefore that the spectra of these difference series have no peaks. And indeed, they all display the same pattern: a concentration of spectral mass at the frequency zero (which reflects their smoothness), yet no indications of any cyclical movements

whatsoever as observed by the differences calculated with the CENSUS-X-11 procedure. This situation does not change even if a high pass filter is run on the low pass differences. As an example, only one spectrum is therefore presented here in Figure 125 below.

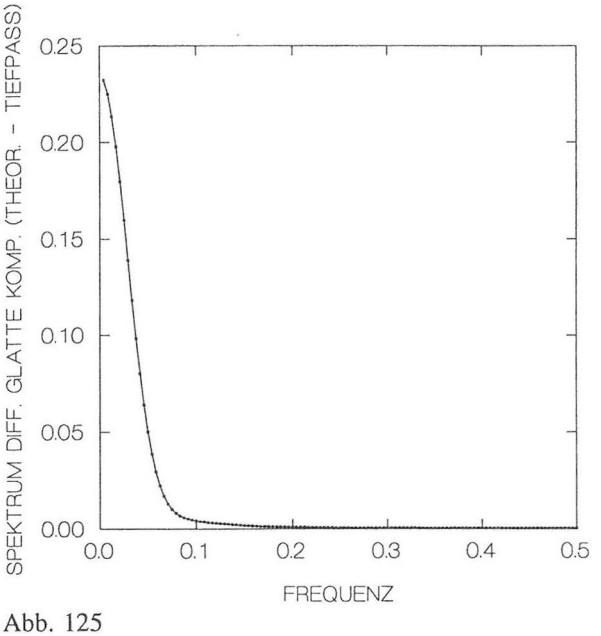


Abb. 125

The observed cyclical movements of the CENSUS-X-11 differences are also to be found in the estimated smooth components of the four series if a high pass filter is run on them, as may be seen by examining the Figures 126-129:

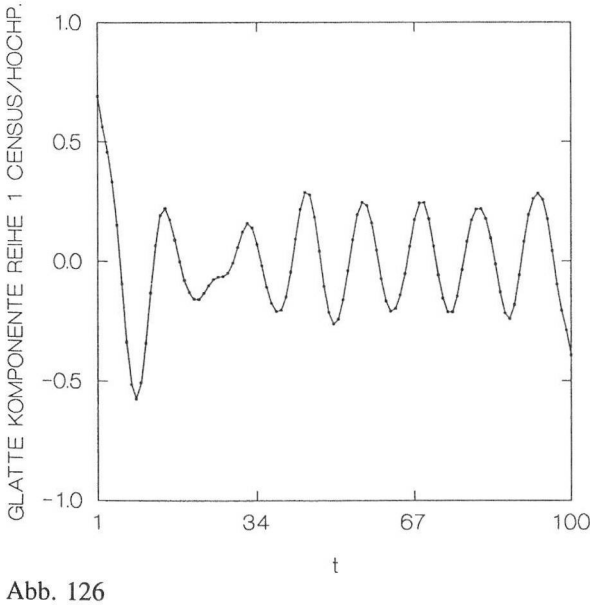


Abb. 126

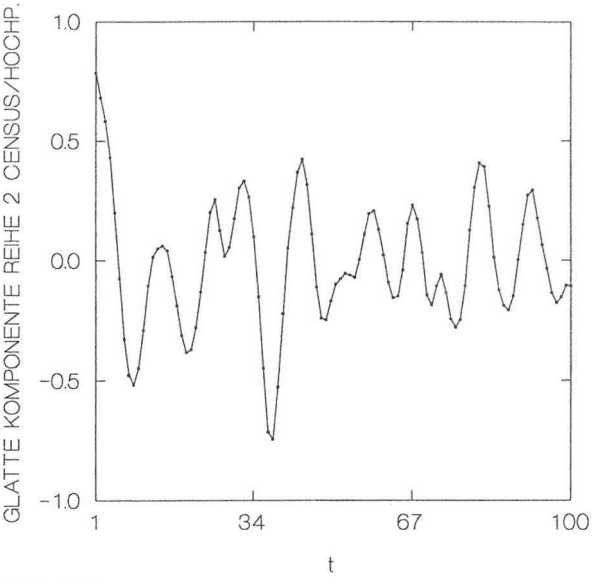


Abb. 127

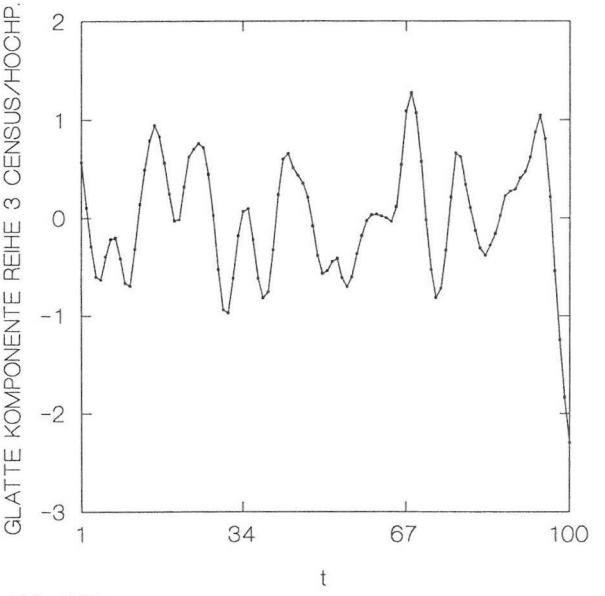


Abb. 128

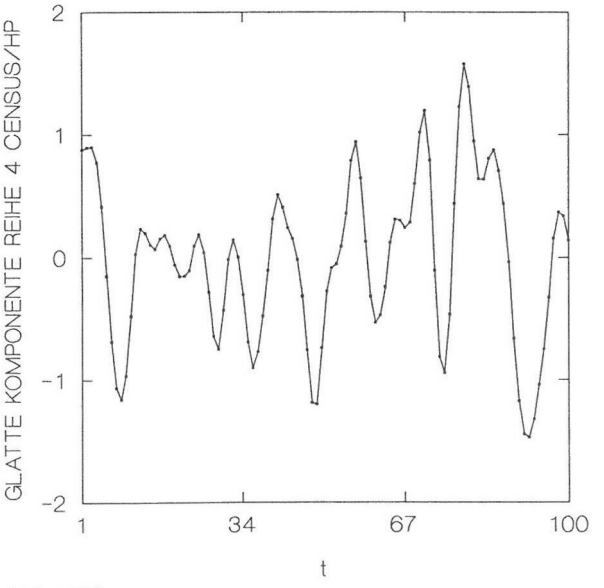


Abb. 129

Comparing Figure 126 with Figure 115, Figure 127 with Figure 116, Figure 128 with Figure 117 and finally Figure 129 with Figure 118, it is easily observed that the patterns of the respective pairs are mirror-images with respect to the zero-line. For illustration, the series pair taken from Figures 126 and 115 is presented in Figure 130:

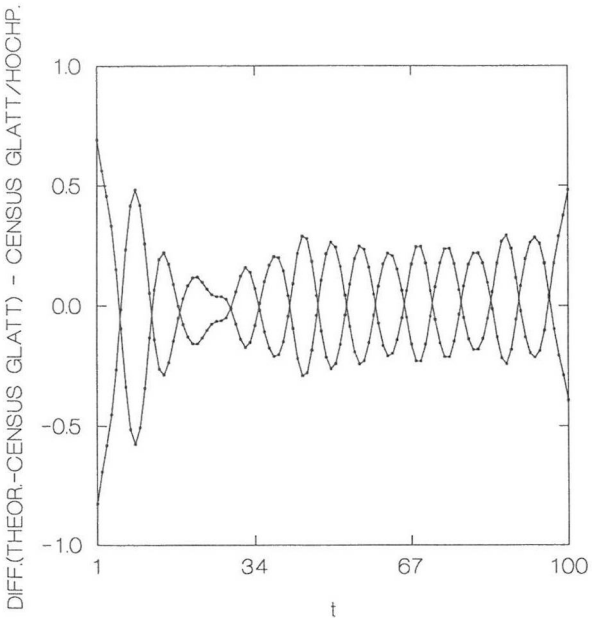


Abb. 130

That the above mirror-image property is not just a matter of chance here, but holds in general, is due to elementary filter theoretical relationships.

The spectra of the high pass filtered smooth components of the CENSUS-X-11 procedure contain therefore exactly the same information as the spectra of the difference-series, that is they exhibit the same cycles mentioned above.

Hence, we conclude:

The smooth components that are derived by the CENSUS-X-11 procedure contain short term cyclical movements which are induced by the procedure itself. The CENSUS-X-11 procedure exhibits a Slutzky-Effect.

For an exposition on the reason for this effect see the discussion of the amplitude functions of the CENSUS-X-11 procedure presented in section 6 of this chapter.

The cyclical movements of the smooth components of the CENSUS-X-11 procedure are not to be regarded as being a simple cosmetical error. If one uses them to calculate *growth rates against the previous period, hence here against the previous month*, as is quite commonly done in practice, then these movements show up also in the growth rates. Consequently, a large risk of deducing false conclusions about the general tendency of such a series is therefore existent. Figures 131-134 display the growth rate paths of the four series that are generated by the smooth components of the CENSUS-X-11 procedure.

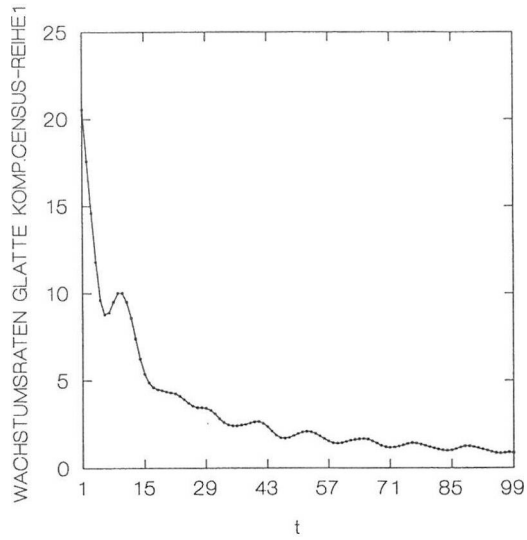


Abb. 131

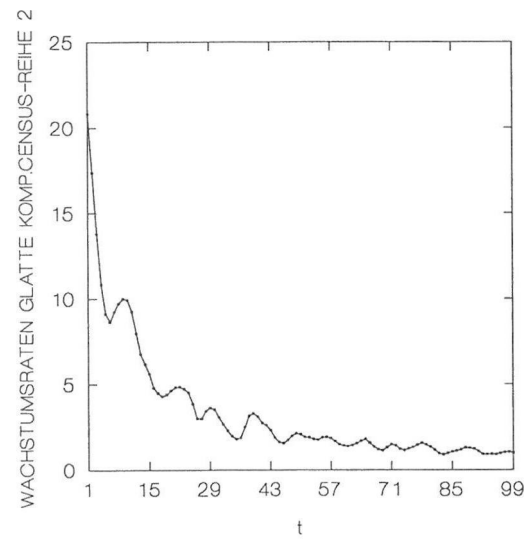


Abb. 132

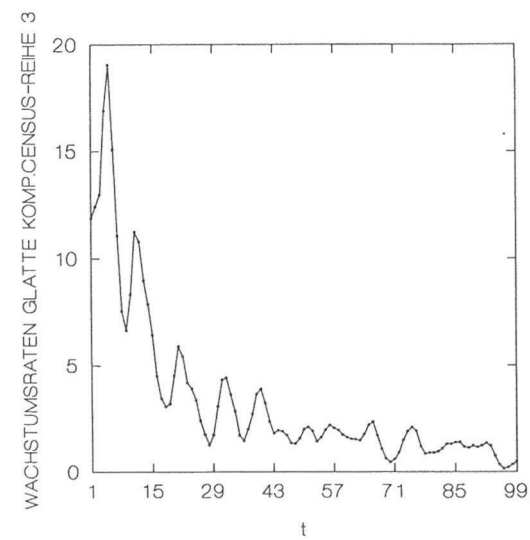


Abb. 133

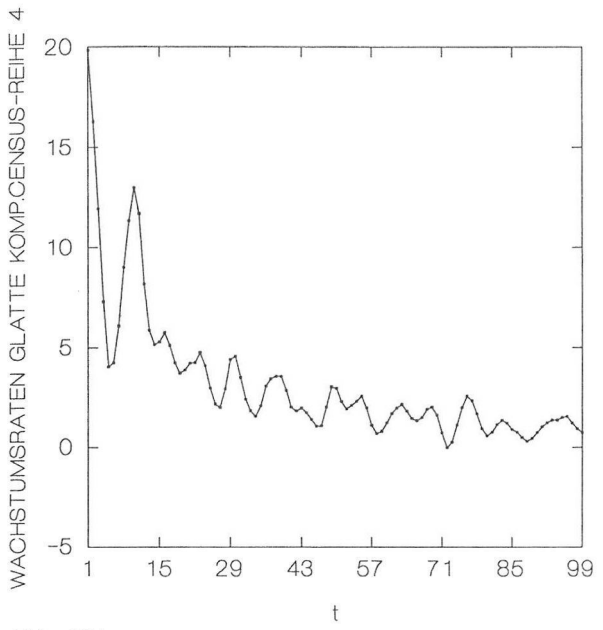


Abb. 134

These growth rates oscillate around the true rates, which exhibit a decreasing monotonic path, given a linear trend. Figure 135 demonstrates these oscillations illustratively for series 4.

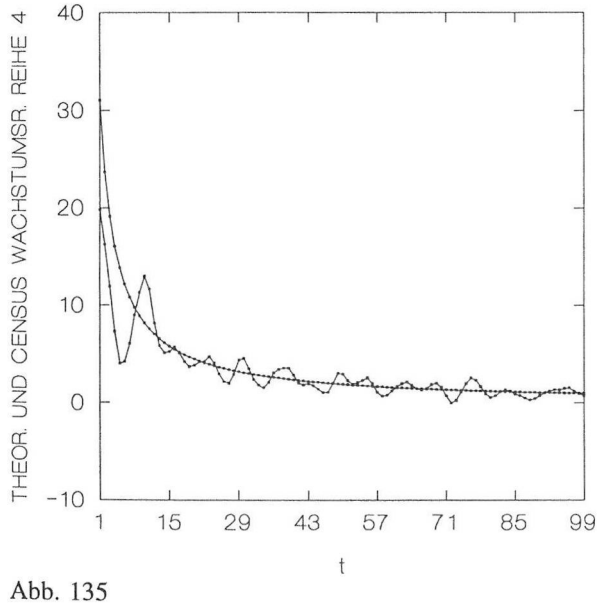


Abb. 135

Contrary to the above, the growth rates of the smooth component obtained by a low pass filter exhibit decreasing monotonic paths, as illustrated in Figure 136 below for all four series together with the actual growth rates.

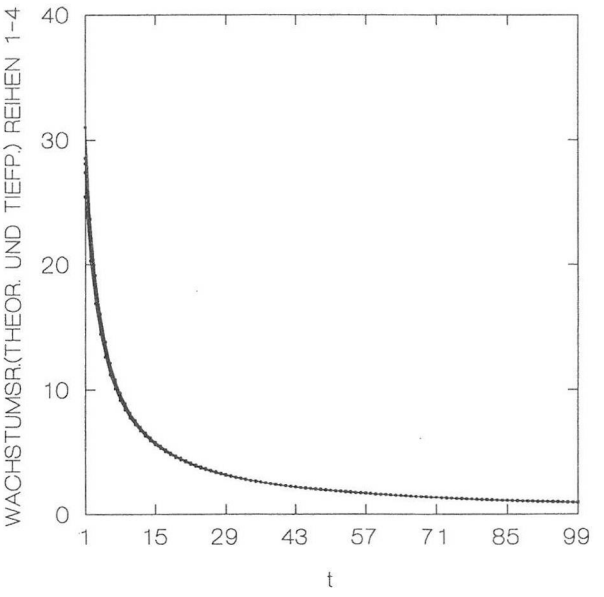


Abb. 136

As examination of Figure 136 shows, these rates may not be optically differentiated from the true rates.

The spectra of the high pass filtered growth rates of the series adjusted by CENSUS-X-11 are shown in Figure 137:

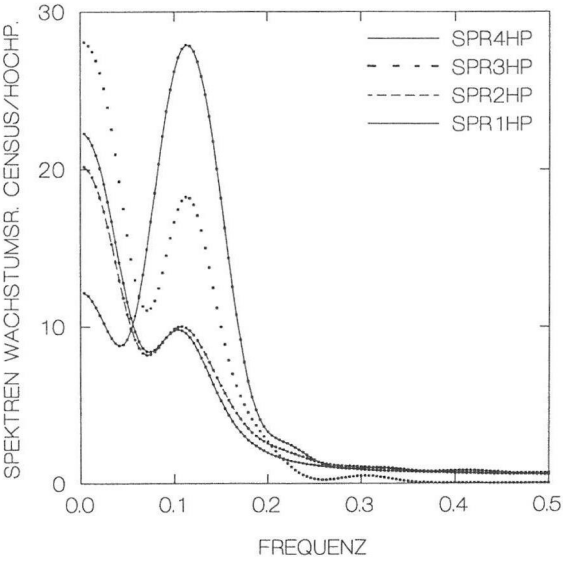


Abb. 137

All four spectra confirm and emphasize the above mentioned grahical impressions. The procedure induces a cycle of 8-9 months, which is especially accentuated for the series 3 and 4.

The short term oscillations of the smooth components according to CENSUS-X-11 not only show up in the growth rates against the previous period, but they may also show up in the corresponding annual growth rates. Figure 138 demonstrates the true annual growth rates of a linear trend

along with the respective rates for the smooth component according to CENSUS-X-11 and the low pass filter.

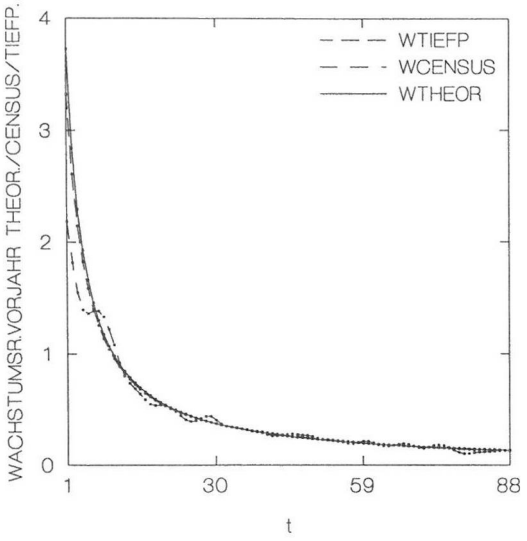


Abb. 138

Whereas the growth rates of the low pass filters hardly differ from the true rates of the simulated series, the growth rates obtained by the smooth components of the CENSUS-X-11 procedure are characterized once again by an oscillating pattern. The oscillations are not as strong as those found for growth rates based upon previous months, which is due in this simulation example to the supposed linear trend.

Finally, we consider the CENSUS-X-11 component of the series presented in Figure 139. In principle, this series was already examined above in chapter II, section 5.3. Due to the fact that the available CENSUS-X-11 program can analyse a maximum of only 240 values, this series was shortened beforehand by 60 values. The model of this series was stated above as:

$$X_t = 3000 + 0.9t + 0.2t^2 + 1000\ (\sin 2\pi/12 + \varepsilon_t)\ ,$$

with

$$\varepsilon_t \sim N(0,2)$$

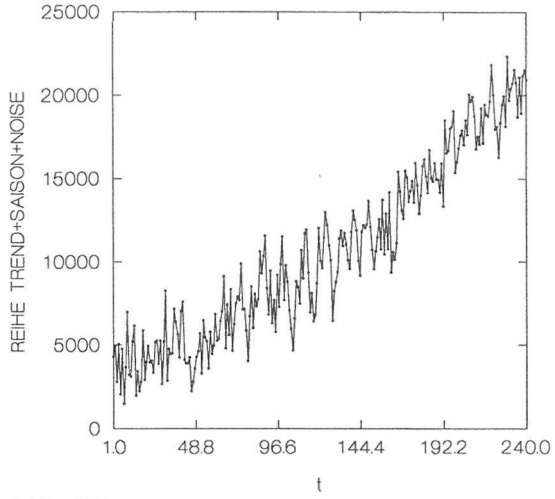


Abb. 139

Let us now estimate the smooth component of this series with the aid of the CENSUS-X-11 procedure, which follows a second order polynomial here. In so doing, one obtains a strongly oscillating series around the true smooth component. This may be seen in Figure 140 below.

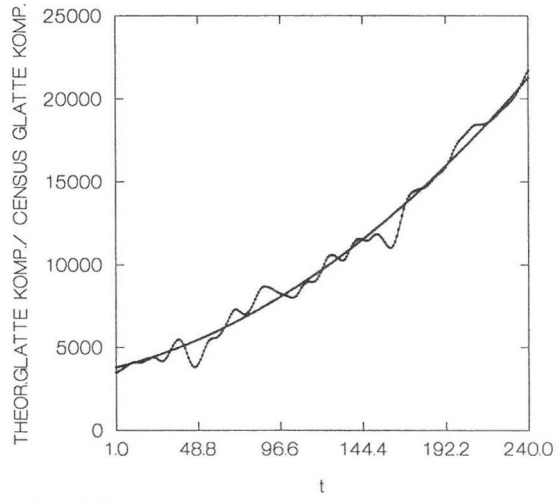


Abb. 140

This "smooth" component would seem to be fully useless for diagnostic purposes. The same holds for the markedly cyclical behaviour of the growth rates (cf. Figure 141):

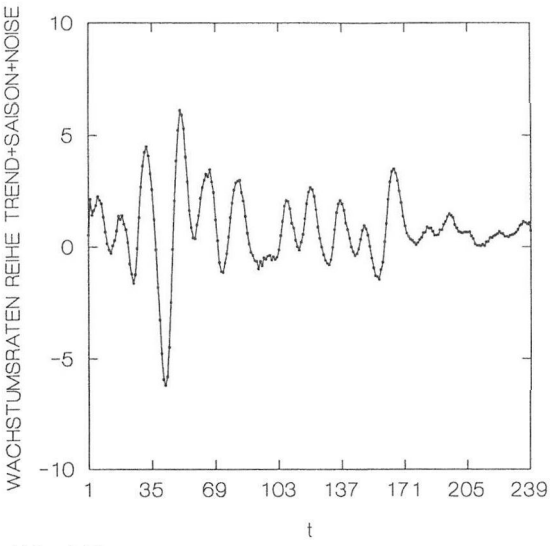


Abb. 141

Looking at the spectrum of these growth rates, it is observed that they exhibit a periodicity of circa 18 months (cf. Figure 142):

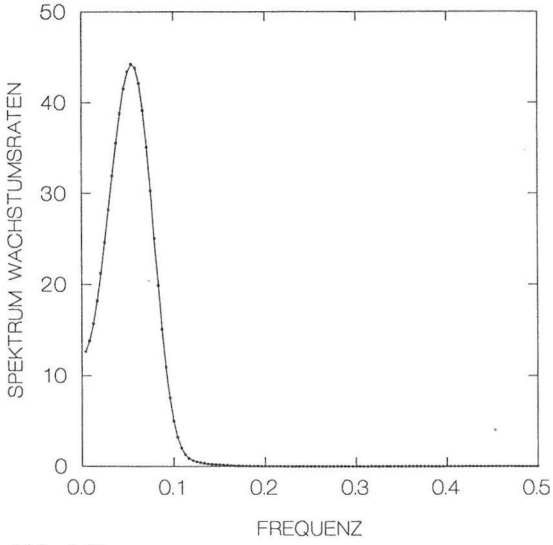


Abb. 142

In contrast to CENSUS a low pass filter generates acceptable results. The true smooth component and the component obtained by a low pass filter differ from one another only slightly (cf. Figure 143):

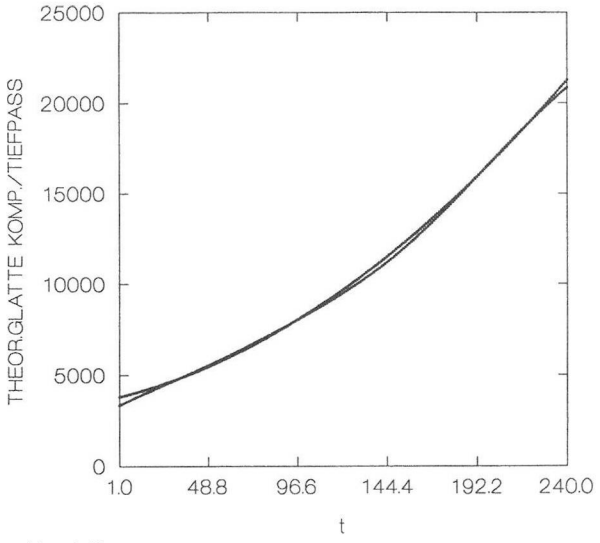


Abb. 143

The unsatisfactory results of the CENSUS-X-11 procedure may be explained quite easily. Considering the adjusted series, one is dealing with a very "dirty" one, which has been constructed so that in the low frequency domain the concentration of the spectral mass originating *not* from the smooth component but rather from the sinusoidal (i.e. seasonality) and from noise is extraordinarily high. This becomes evident if one examines the (estimated) spectra of the three series components, which are partially presented below up to the second seasonal frequency (i.e. up to the frequency 2/12 = 0.1667). The labels below have the following meanings:

SPGK : spectrum of the trend polynomial $3000+0.9t+0.2t^2$
SPNOISE : spectrum of noise 1000 ε_t
SPSINUS : spectrum of 1000 $\sin(2\pi/12)$

FREQUENCY	SPGK	SPNOISE	SPSINUS
0.004166667	1086793000	6526690	6546031
0.008333334	1008096000	6562007	6579868
0.0125	888678400	6595290	6610358
0.01666667	743644800	6595407	66605499
0.02083333	589780500	6531433	6533556
0.025	442551700	6381506	6374255
0.02916667	313724600	6139848	6129918
0.03333334	210074200	5820196	5833685
0.0375	133321100	5454762	5551754
0.04166667	81098960	5088893	5377269
0.04583333	48525940	4772599	5415260
0.05	29874610	4550844	5760439
0.05416667	19917740	4454729	6472215
0.05833333	14710750	4495451	7553007
0.0625	11787440	4662242	8936130
0.06666667	9915929	4924546	10487880
0.07083333	8644649	5237796	12025150
0.075	7856216	5551391	13345860
0.07916667	7468170	5817185	14265850
0.08333334	7318606	5996814	14653820
0.0875	7192830	6066666	14455860
0.09166667	6909065	6019878	13704040
0.09583333	6388157	5865449	12507160
0.1	5667964	5625042	11026570

0.1041667	4864193	5328386	9443570
0.1083333	4106664	5008245	7926344
0.1125	3485385	4695778	6603966
0.1166667	3028006	4416885	5552296
0.1208333	2710255	4189857	4792982
0.125	2485152	4024375	4303341
0.1291667	2311784	3921766	4032556
0.1333333	2169970	3876247	3918972
0.1375	2057952	3876826	3904148
0.1416667	1979788	3909528	3941324
0.1458333	1932764	3959573	3998082
0.15	1902578	4013244	4054658
0.1541667	1867831	4059265	4100217
0.1583333	1809713	4089638	4129118
0.1625	1720489	4099991	4138528
0.1666667	1606035	4089561	4127628

As can easily be seen, in certain frequency domains the spectral mass which may be attributed to the seasonal and noise, is significantly larger than the corresponding spectral mass due to the smooth component. Due to the unfavorable amplitude function of the smooth component of the CENSUS-X-11 procedure, both at the center as well as at the boundary, the spectral mass is transformed into cyclical movements which are erroneously added to the smooth component. In other words, the CENSUS-X-11 procedure is not capable of correctly separating the series components.

It is to be stressed here, that the good low pass-results presented above can only be obtained if one uses a low pass filter with *extremely short pass bands*. Only then is it possible to eliminate that part of the spectral mass which does not belong to the smooth component. Therefore in the case examined above, the frequency domain [0, 0.005] was chosen as the pass band.

The series just analysed is certainly not typical for economic series. However, low pass filtering does demonstrate first of all, that it is not always appropriate to employ a standard pass band (for "normal" economic series e.g. the pass band [0, 0.007]). Secondly, the above analysis demonstrates the *flexibility* of a filter-theoretical approach, a flexibility that is not obtainable using a standard approach such as the CENSUS-X-11 procedure.

Literature

1) STIER W., EDEL K., 'Eigenschaften von Saisonbereinigungsverfahren im Frequenzbereich', in: K.A. Schäffer, K. Edel, P. Miebach (Eds.), Probleme von Saisonbereinigungsverfahren, eine vergleichende Studie, forthcoming
2) STIER W., Verfahren zur Analyse saisonaler Schwankungen in ökonomischen Zeitreihen, Berlin-Heidelberg-New York 1980
3) SCHÄFFER K.A., 'Beurteilung einiger herkömmlicher Methoden zur Analyse von ökonomischen Zeitreihen', in: W. Wetzel (Ed.), Neuere Entwicklungen auf dem Gebiet der Zeitreihenanalyse, Göttingen 1970, pp. 131-164

IV. Applications of the CENSUS-X-11 Procedure for Time Series with Price Quotations and Recommendations for Future Proceeding

1. Evaluation of the CENSUS-X-11 Adjusted Time Series

If one examines time series that have been seasonally adjusted by the CENSUS-X-11 procedure, then it is observed that not all of the seasonal frequencies have always been entirely filtered out. Frequently, however, this may not be directly observed by simply looking at the series. Only after a detailed examination with the aid of spectral analysis - implementing on appropriately high passed filtered seasonally adjusted series - one can observe such inefficient adjustment results. For example, Figure 144 below illustrates the spectra of the total index:

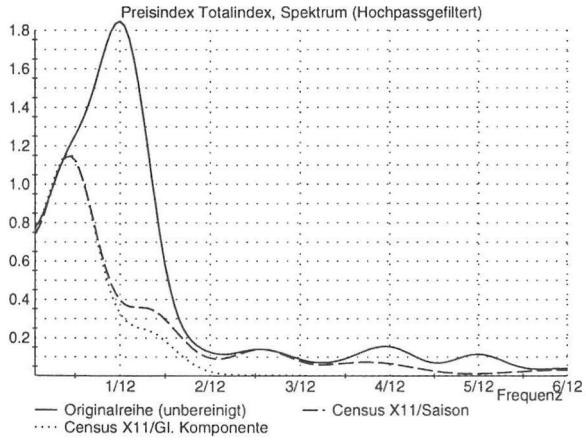


Figure 144

This graph demonstrates that even the first seasonal frequency is *insufficiently filtered out* by both the seasonal adjusted as well as the smooth component. For the seasonal adjustment (yet not for the smooth component) this is further the case for the second, third, fourth and sixth seasonal frequency.

Taking a look at other series, this property of the CENSUS-X-11 procedure is even more distinct. e.g. for the series DIENSTL (services) and MIETE (rents), whose spectra (for the high pass filtered original and the seasonal adjusted series along with the high pass filtered smooth component) are presented below in Figures 145 and 146.

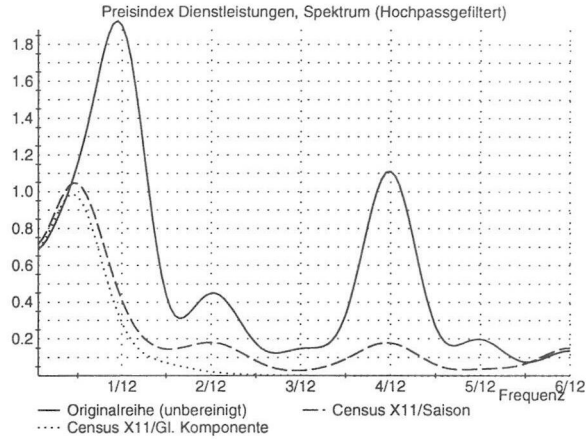


Figure 145

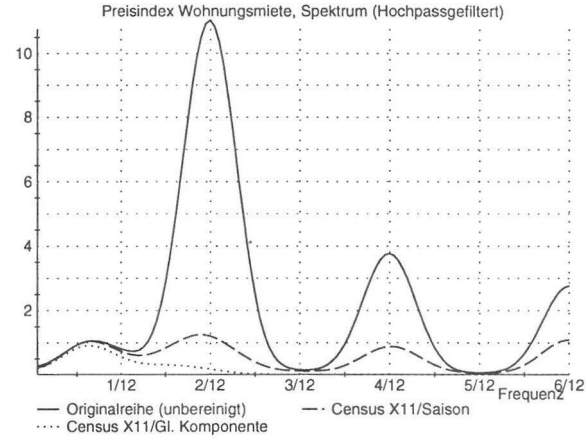


Figure 146

In particular, both series are characterized by an insufficient attenuation of the quarterly cycle. However, also the half-yearly cycle is insufficiently attenuated, which especially dominates the series MIETE (rents).

This "tracing-property" at the boundary (discussed below in section 3 of this chapter) of the smooth CENSUS-X-11 component is also confirmed for end series (this concept is also discussed in section 3 following) that have been seasonally adjusted with this procedure. This is once again not dependent upon whether the total index is first adjusted and then the respective inflation rates are calculated (i.e. the growth rates against the previous year) or vice versa. The results of both the above mentioned possibilities are presented below in Figure 147, whereby the dashed vertical lines designate the beginning of the actual end.

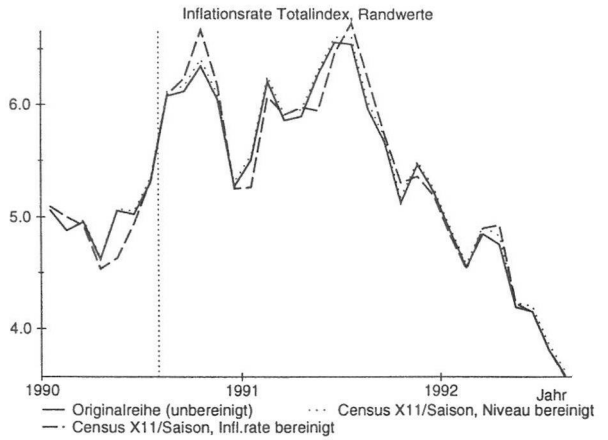


Figure 147

For both approaches there is practically no indication of a seasonal adjustment at the boundary.

With regard to this, the general question arises as to whether first of all the standard series should be seasonally adjusted with the CENSUS-X-11 procedure, in order to calculate the (seasonally adjusted) growth rates, or if a reversed approach would be more advisable. Considering this question from the perspective of filter theory, then both approaches represent two filters in *cascade-connection*, i.e. the filter output of the first filter is the filter input of the second. It can be shown that the output of the total filter is *independent* of the order of the applied filter, given that *both* filters are *linear*. One may easily convince oneself of this by taking an arbitrary time series, applying sequentially a moving five term and a moving three term average filter (both are linear filters). Changing the sequence of these filters has no influence upon the final result. This situation changes immediately, however, if *one* of the two filters is nonlinear. This may also be easily controlled for, e.g. by first calculating growth rates for an arbitrary time series (this is a nonlinear filter) and then proceeding with a applying a moving average. If the sequence of the filter operations is now reversed, that is, if one smooths the series first and then calculates the growth rates, it can be seen that the final result is dependent upon the sequence of the filters.

Since the problem at hand involves *two* nonlinear filter operations, it is clear from the start that the choice of the approach taken will affect the final results. It is, however, basically not possible to say *generally* which filter sequence is superior. Experience seems to show rather that any decision with regard to which approach is more favorable is contingent upon the objective of the analysis, upon the type and magnitude of the resulting differences, and above all upon the observed series. In the following, we discuss these aspects in some detail using real series.

If a *prospective* objective is the main emphasis of analysis, then the filter sequence should be determined according to its boundary properties. As shown above, a growth rate filter inflates the noise component considerably. Due to the fact that the amplitude functions of the CENSUS-X-11 procedure exhibit *unfavorable boundary properties* - both for the seasonal adjustment as well as for the smooth component - seasonal adjustment resp. determining the smooth component will additionally increase the variance of the noise component. Taking this into consideration, it appears to be advisable when implementing the CENSUS-X-11 procedure to first adjust the standard series and then calculate the growth rates. That this proposal is adequate at least for some time series may be seen by the example of the series TOTALIND. If one compares the pattern of the seasonally adjusted end values for both alternatives (cf. Figure 147), then it becomes evident that a seasonal adjustment of TOTALIND in which the growth rates are calculated in a second step is more favorable than the other way round since calculating growth rates first, not only the growth rates are traced at the boundary (see the discussion above), but additionally the fluctuations of the growth rates are enhanced. Comparison of both alternative approaches with regard to the smooth component (cf. Figure 162 in this chapter) results basically in a similar evaluation.

Yet a definitive decision with respect to this series does not seem possible, given that a *retrospective* objective is the basis of the analysis. Figure 148 below contains both alternative smooth components of the growth rates of the series TOTALIND.

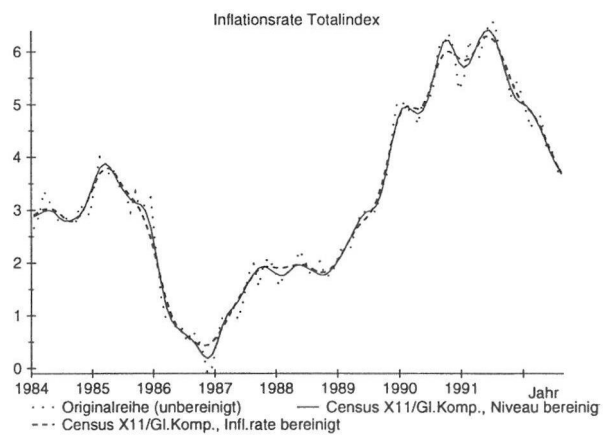


Figure 148

Looking at the smooth component for both alternatives, one observes that they are quite similar over their entire time period. Any attempt to differentiate them accordingly would be purely subjective.

Figure 149 presents both alternatively seasonally adjusted growth rate series of TOTALIND, as well as the inflation rates of the unadjusted series.

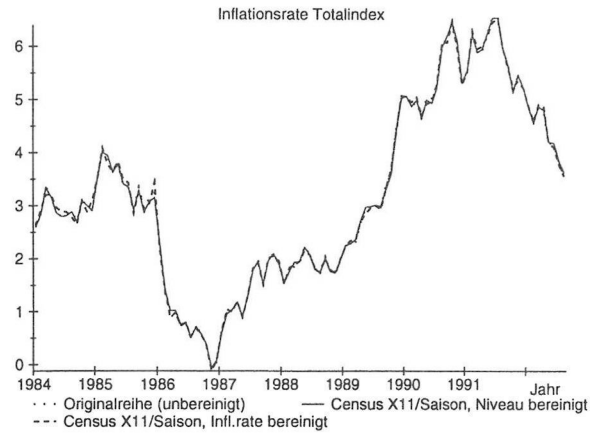


Figure 149

Here also the apparent differences are rather small so that a simple comparison of the alternative series does not offer sufficient evidence to base a selective decision upon.

Now let us take a quick look at the growth rates of the services series DIENSTL. Figure 150 below illustrates the results obtained for the smooth components.

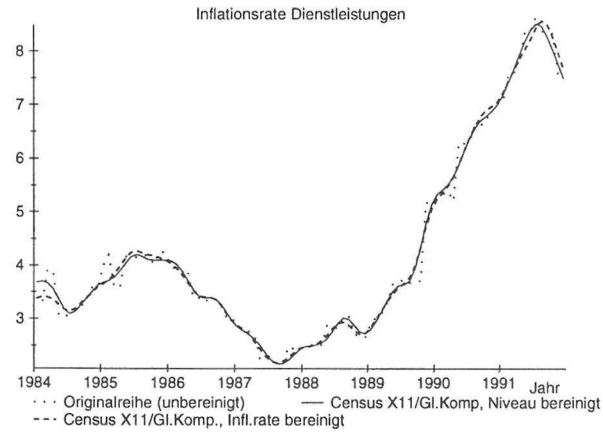


Figure 150

One observes throughout a strong (positive) correlation between both smooth components in which usually only minimal differences are present. In particular, the filter sequence "growth rate/smooth component" exhibits a phase shift following the year 1991, which could lead one possibly to prefering the reversed procedure. For the seasonally adjusted growth rates, one obtains Figure 151:

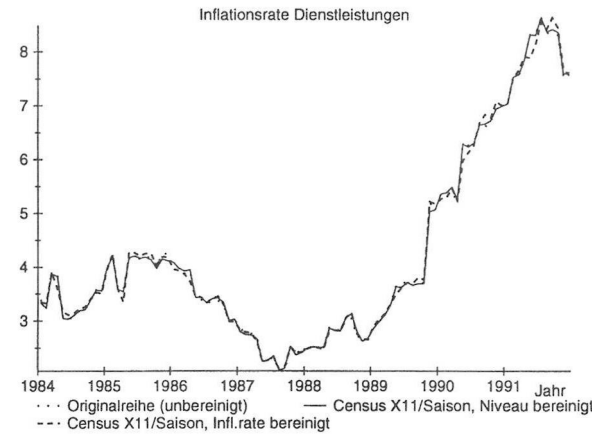


Figure 151

As was the case for the above series, a comparison also offers no criteria for prefering one filter sequence over the other.

Finally, let us examine the series MIETE (rent). For the smooth component we obtain Figure 152:

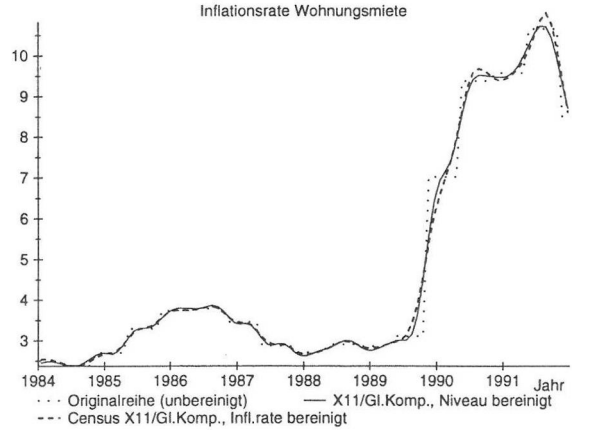


Figure 152

Approximately after the year 1990 differences emerge. The filter sequence "growth rates/smooth component" generates at the series end an "oscillating overshooting" of the smooth component, so that one would tend to prefer the alternative filter sequence. The same conclusion occurs comparing both of the seasonally adjusted growth rates, as may be seen in Figure 153 below.

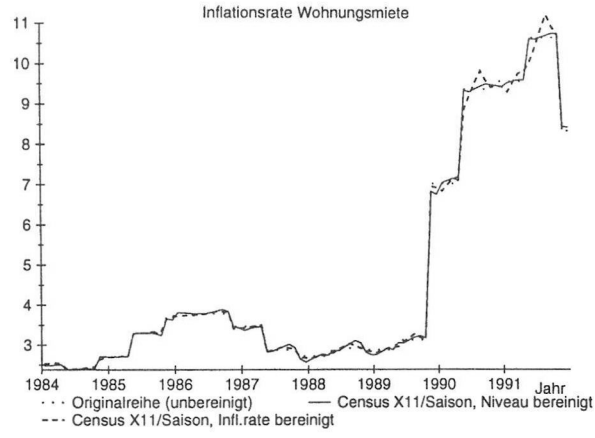


Figure 153

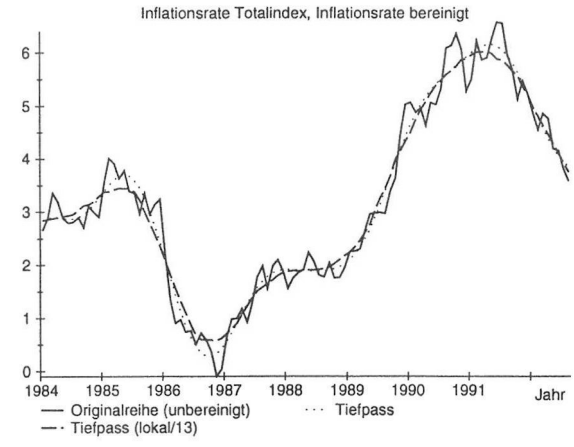


Figure 155

The above presented illustrations exemplify the fact that the question as to which filter sequence is preferable given a retrospective objective of the analysis may not always be decided unequivocally if the analysis uses the CENSUS-X-11 procedure. All the same, the above results do seem to suggest that a filter sequence "adjustment of the level/ growth rates" is preferable to the opposite sequence. In general, however, a clear decision with regard to the filter sequence seems to be more feasible if the analysis follows a prospective objective.

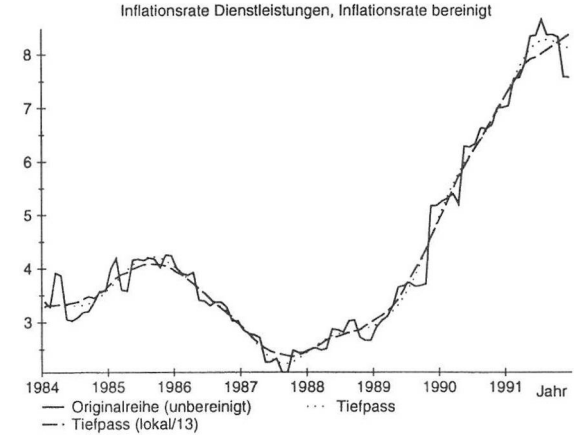


Figure 156

2. Applications of Local Low Pass Filters

Figure 154 below illustrates the calculated inflation rates by using a local low pass filter upon the total index.

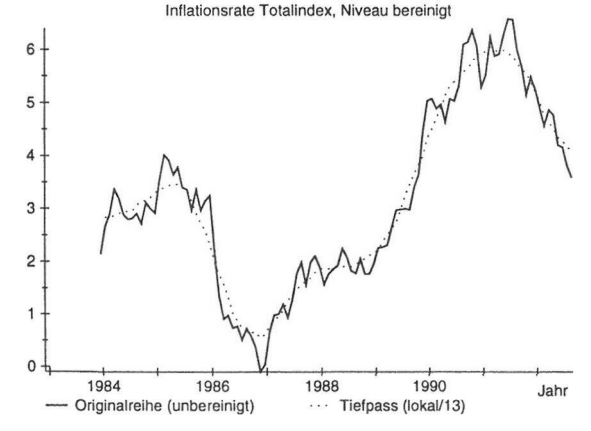


Figure 154

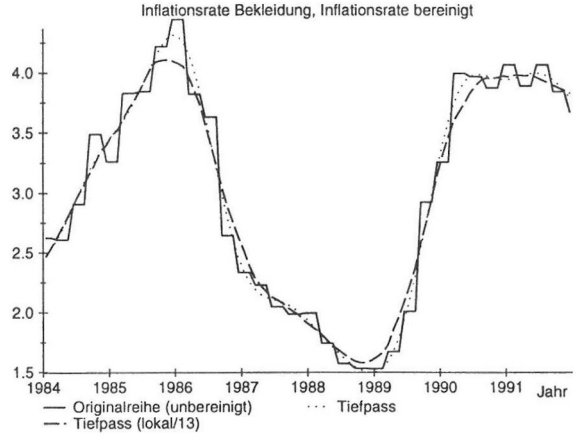


Figure 157

The following four Figures 155-158 present a comparison of the respective global and local low pass filters for the growth rates against the previous year (rates of inflation) of the time series TOTALIND, DIENSTL, KLEID and MIETE.

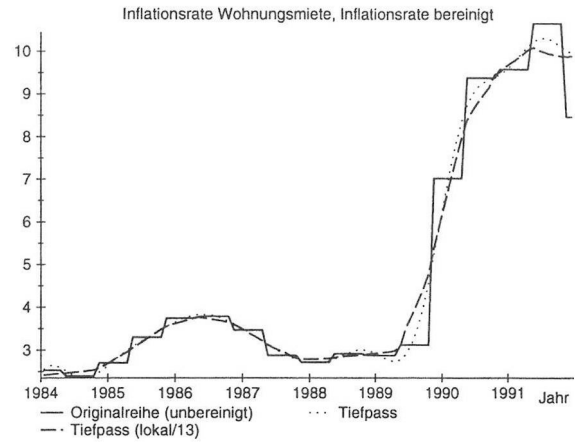


Figure 158

As illustrated by the above graphs, there are no systematic differences between both filters. The local low pass filter shows, however, a somewhat more volatile behaviour and displays somewhat smaller extremes than the global low pass filter. One of the reasons for this is the fact that a local filter processes *time dependent* information (the frequency information of successive supporting spans), whereas a global filter processes information from exactly *one* supporting span. Since the variances of the successive supporting spans are smaller than the variance of the total series, it is to be expected that the variance of the locally filtered series is altogether smaller than the globally filtered series. This explains the more moderate fluctuations observed.

If one estimates the spectra of the previous global and local low pass filtered time series (after a high pass filtering), then one finds that the spectra of the locally filtered series are below the spectra of the globally filtered series throughout. This is demonstrated illustratively in the following Figure 159 for the time series TOTALIND, whereby in addition also the estimated spectrum (also after a high pass filtering) of the smooth component according to the CENSUS-X-11 procedure is included.

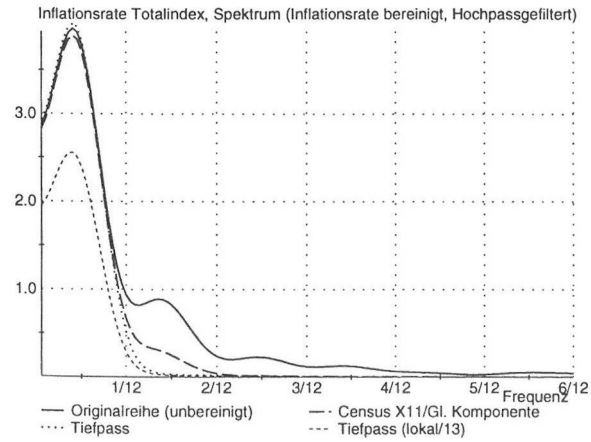


Figure 159

(Furthermore, from this figure it can be observed that both low pass filters achieve a better adjustment in the high frequency domain than achievable with the CENSUS-X-11 procedure.)

In accordance, the variances of the (high pass filtered) global and local low pass filter-outputs are:

Series Variance	*Global LP*	*Local LP*
TOTALIND	0.247	0.156
DIENSTL	0.0353	0.0165
KLEID	0.0504	0.0239
MIETE	0.193	0.0959

3. Boundary Properties of the Global and Local Low Pass Filter and the Smooth Component of the CENSUS-X-11 Procedure

For practical purposes, especially for current diagnosis, the *boundary properties* of a seasonal adjustment procedure are quite important. It is possible to derive the boundary properties of a specific procedure by dropping values one by one and then successively elongating again the series by one. Thus, an *adjusted boundary series* may be constructed, i.e. a series which consists *only* of (adjusted) boundary values and therefore is briefly named "*boundary series*".

Figure 160 shows both end series of the inflation rates for the global and local low pass filters, whereby the dashed vertical lines designate the beginning of the boundary.

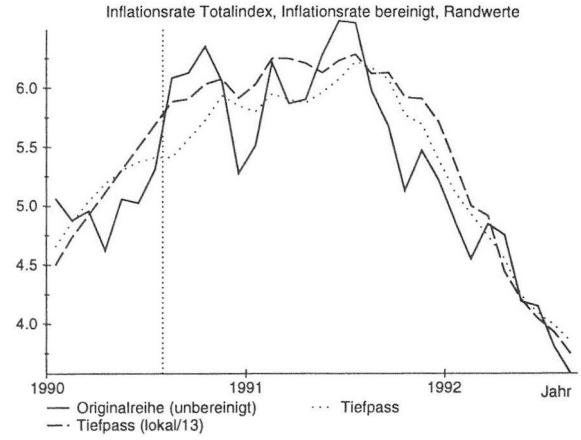


Figure 160

Obviously, both low pass filters have adjusted the end values quite well in contrast to CENSUS-X-11 for which boundary values of the smooth component exhibit the property of *"tracing"* more or less the original time series (with a phase shift). This is quite evident in the following Figure 161 which contains the inflation rates of both the end values of the local low pass filters, as well as the end values of the smooth component of the CENSUS-X-11 procedure:

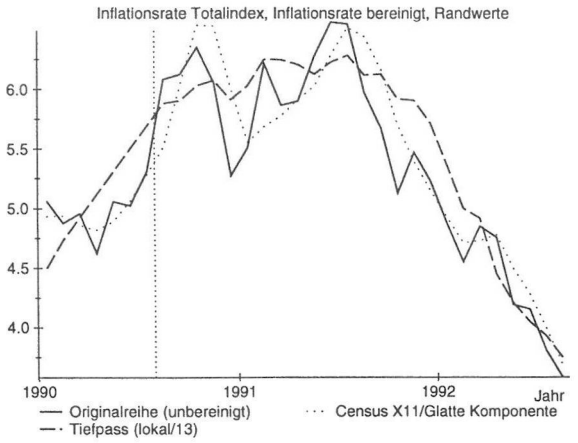


Figure 161

This end value phenomenon is to be found independently of whether the smooth component of the total index is first estimated and then the respective inflation rates are calculated, or whether one primarily calculates the inflations rates and then estimates the smooth CENSUS components. Both approaches are illustrated together in Figure 162.

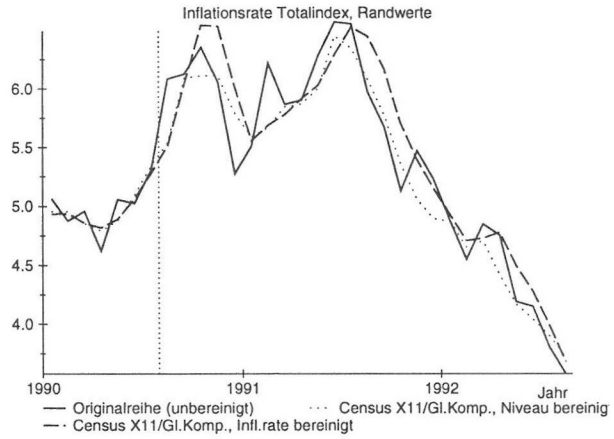


Figure 162

For both approaches a "tracing" effect is apparent. However, examining the CENSUS-X-11 adjusted inflation rates one further observes a phase of approximately one month.

These conspicuously poor boundary properties of the smooth component (in the time domain) of CENSUS-X-11 are not only to be found in the above series. They should rather be considered as a *feature* of this procedure, since they are founded upon the fully insufficient properties of the boundary amplitude function of the smooth component as discussed above in detail in chapter III. Consequently, for practical purposes the following conclusion emerges:

The boundary values of the smooth component obtained by the CENSUS-X-11 procedure must be considered to be unusable and unsound for diagnostic purposes.

That the local low pass filter also produces acceptable adjusted values for the boundary may finally be seen by a comparison of local final adjusted values and the original boundary values, whereby the latter exhibit a slight phase shift (cf. Figure 163):

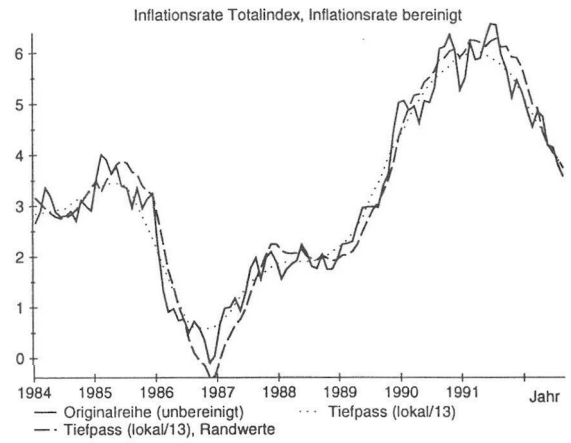


Figure 163

4. Summary

The most important results of this study may be summarized briefly as follows:

Analysis of the CENSUS-X-11 procedure with regard to the properties of the two *boundary amplitude-functions* - for seasonal adjustment and for the smooth component - has shown that both exhibit *especially unfavorable* behaviour. This has *practical consequences*. The boundary values of a time series are not seasonally adjusted, nor are the smooth components of these values determined. Indeed, in both cases one observes practically a *tracing* of the movements of the original values, combined with an inflation of the variance. Consequently therefore, it follows that the obtainment of a *prospective* objective of a seasonal adjustment is *not realizable* with CENSUS-X-11. The fact that both phase functions exhibit a favorable behaviour - especially in the low frequency domain - which for *boundary series* results in *smaller phase shifts*, does not affect this viewpoint. As has been demonstrated, this is simply a consequence of the two fully insufficient boundary amplitude-functions.

For current diagnostic purposes (prospective objective), with respect to both the seasonal adjustment as well as the smooth component, the CENSUS-X-11 procedure is inappropriate.

Somewhat more favorable are the properties of the CENSUS-X-11 procedure given that a *retrospective* objective is being followed. The *amplitude function* for the *saisonal adjustment* approximates the *ideal function very closely*, and also the *phase function* exhibits practically a *zero phase*. Unfortunately, however, even given a retrospec-

tive objective the *smooth component* remains problematical, as could be demonstrated, due to a *Slutzky Effekt.*

For analyses based upon a retrospective objective (e.g. for historical economic analyses), the CENSUS-X-11 procedure may be applied successfully. However, the smooth component of this procedure may be utilized in such studies without hesitation only if one regards the Slutzky Effect to be insignifcant or unimportant for the topic at hand.

If in addition to the level of a time series growth rates (e.g. compared to the previous year) are also of interest, then it is advisable to use the sequence "seasonal adjustment/ growth rates" resp. "smooth component/growth rates" when implementing CENSUS-X-11. The reversed filter sequence would cause an undesireable "roughing" of the series in particular at the boundary, which is magnified by the unfavorable CENSUS-X-11 boundary properties. However, even for this filter sequence one must remember that the adjusted growth rates at the boundary are not suitable for diagnostic purposes. The above mentioned "tracing effect" of the time series by the boundary values is also present here.

Since for topical diagnostic purposes the smooth component of a time series is preferable to the seasonally adjusted time series - the latter still contains the noise component, which complicates or makes impossible any diagnosis of the current development (is there an upward- or downward- or stagnating tendency?) - the question arises as to what other *alternatives* exist other then the inefficient smooth components of the CENSUS-X-11 procedure. As an alternative, the proposal made here is to apply a *low pass filter*, which may be implemented either as a *global* or as a *local* low pass.

Both filters permit the determination of the smooth component at the boundary, that is, they may be applied given a prospective objective and thus for current diagnostic purposes.

The results of a global low pass filter are somewhat smoother than of a local one, however, it exhibits the *disadvantage* that by *updating*, i.e. the inclusion of new time series values, previously filtered values may change. This is not the case for a local low pass filter. Given a supporting span of the length of 13 months, only the last six filtered values may change. The series of the *end values* is characterized - similar to the CENSUS-X-11 procedure - by a *small phase shift.*

Since the amplitude properties of both low pass filters are favorable, the sequence of filtering is *in principle arbitrary* when adjusting growth rates. However, since the amplitude functions of both filters can not be an exact rectangle function - the reasons herefore where presented above - it is advisable to apply the filter sequence growth rate /low pass filter.

Finally, it should generally be mentioned as already above in chapter II, that alternatives to the CENSUS-X-11 procedure do exist such as the procedures by KITAGAWA-GERSCH, PAULY, SCHLICHT etc. However, to date there exist no studies concerning the properties of the amplitude- and phase functions of these approaches. Of particular interest for practical implementation, no studies exist with regard to the especially important boundary-problem. Only one comparative study (in the time domain) by SCHÄFFER exists dealing with the boundary properties of the KITAGAWA-GERSCH procedure in comparison to the CENSUS-X-11 procedure. In consideration of the boundary properties presented in our above analysis, it should not be of any surprise, to find that the KITAGAWA-GERSCH procedure studied by SCHÄFFER proved to be preferential [1].

Literature

1) SCHÄFFER K.A., 'Probleme bei der Analyse der Arbeitslosenreihe', in: L. Reyher, J. Kühl (Eds.), Resonanzen, Arbeitsmarkt und Beruf - Forschung und Politik, Nürnberg 1988, pp. 385-400

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- Die Revision des Landesindexes der Konsumentenpreise, Konzept für einen neuen Landesindex, Bern 1993.
- Die Revision des schweizerischen Grosshandelspreisindexes: Grobkonzept für einen neuen Produzentenpreisindex, Bern 1990.
- Die Revision des schweizerischen Grosshandelspreisindexes: Konzept für den neuen Produzentenpreisindex, Bern 1992.
- Verbrauchserhebung 1990: Ausgaben und Einnahmen der privaten Haushalte, Bern 1992.

In addition to the permanent publication of results, the following publications will be published in 1995:

- Grundlagen des revidierten Landesindexes der Konsumentenpreise.
- Grundlagen des revidierten Produzentenpreisindexes.

The publications above contain a detailed description of bases and methods regarding the revised price statistics.

- Long series of the consumer price index.
- Long series of the producer price index.

The publications above contain time series resulting from the linkage of the revised consumer price index respectively producer price index and the corresponding previous time series.

The objective of the following study, commissioned by the Swiss Federal Statistical Office (Bundesamt für Statistik, BFS), is to present a survey of seasonal adjustment procedures for economic time series reflecting the current state of the art as well as the practical implementation of the analyzed methods in official statistics.

In addition to a general discussion of seasonality and the design of seasonal adjustment procedures, the study analyses, in particular, the widely used CENSUS-X-11 procedure. The analysis centers around the suitability of the CENSUS-procedure for adjusting price quotations in Switzerland.

This study offers a valuable contribution with respect to both theoretical and practical issues in the complex topic of time series in price statistics.

This study was originally published in German by the Swiss Federal Statistical Office in 1993.

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