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# Small Area Estimation in the Structural Survey

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Small Area Estimation in the Structural Survey: Phase II Work for the Swiss Federal Statistical Office

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# 1 Introduction

In Phase I of this project, several small area estimators of the proportions of active people in the Swiss districts were compared. This was done by means of simulation studies under different setups, namely model-based, design-based but knowing the data generating process and design-based with unknown data generating process and considering the Structural Survey data as the population data. For that purpose, several specific models were fitted, including a linear mixed model (LMM) and a generalized linear mixed model (GLMM). These two mentioned models produced very similar district estimates; then, the LMM was finally selected because of its simplicity of implementation and computational efficiency as compared with the GLMM. The selected LMM model was checked by different means including the mentioned simulation studies and also through model diagnostics, and results indicate that this model is relatively good in terms of predicting activity in the Swiss districts. The selected LMM contains all the auxiliary variables listed in Table 1. More concretely, the model contains random district effects and fixed effects for all the categories of all other variables in Table 1, leaving out the first of them as base reference and including an intercept. Fixed effects were also included for the interactions between `gender` and `age group`, and between `gender` and `civil status`. The estimated regression coefficients are listed in Table 1 of Appendix 2 and we can see that all of these variables are strongly significant and the signs of the coefficients are somehow intuitive.

In the simulation studies of Phase I, the 6 districts will the smallest sample sizes ($<150$) in the Structural Survey data were discarded, and the remaining data in that survey was treated as the population data. Simulations were based on drawing smaller samples from that "population", but taking sample sizes for some districts as small as the district sample sizes in the Structural Survey.

Phase II of this project consists of using the whole data set from the Structural Survey, including all the districts, to fit the model selected in Phase I. The values of the covariates included in the STATPOP data set for all the individuals in the target population are used to predict the proportions of active people in the Swiss districts through the model. In order to assess the reliability of the obtained estimates, mean squared error (MSE) estimates must be also computed. Mean squared errors will be obtained under two different philosophies or approaches, namely the model-based approach (model MSE) and the design-based approach (design MSE). It is important to remark that the interpretation of these two MSEs is different, since the model MSE is the average squared error over all the possible realizations of the (random) population, whereas the design MSE is the average squared error over all the possible samples drawn from a (fixed) population. When the model is correctly specified (in practice this is seldom exactly true but good approximations may be found), the model MSE averaged for a large number of areas should be similar to the average design MSE for those areas, see Theorem 1 (ii) in Appendix 1.

A parametric bootstrap method will be used to estimate the model MSE. The properties of the bootstrap model MSEs as estimates of the model MSE will be analyzed in a model-based simulation study. Additionally, a design-based simulation study considering the Structural Survey data as the true population will analyze the properties of the bootstrap model MSE as an estimator of the design MSE.

A nonparametric bootstrap procedure is proposed for estimation of the design MSE. Simulations cannot be performed to analyze this estimator because the values of the target variable (`active`) for all the population units are not available and hence we cannot perform simulations imitating de exact Swiss sampling design. Thus, design MSE estimates using this nonparametric bootstrap method will be computed using the STATPOP data and compared with the parametric bootstrap model MSE estimates.

Section 2 describes the considered estimators of the proportions of active people. Sections 3 and 4 present the proposed bootstrap methods for estimation of model MSE and design MSE respectively. Section 5 describes the simulation studies performed to analyze the properties of the parametric bootstrap procedure. Section 6 comments on the results obtained using the STATPOP data set. Concluding remarks are given in Section 7.

Table 1: Variables (final)

| active | 1=active 0=inactive |
|---|---|
| district | there are 147 of them |
| Strata1 | 0=Strata with large sampling weight median, 1=otherwise |
| District1724 | 1=District nr 1724 / 0=otherwise |
| age group | 15, $[16,20)$, $[20,60)$, $[60,64)$, 64, $\geq 65$ |
| gender | male (1) / female (2) |
| civil status | 1 = single, unmarried, 2 = married, in a registered partnership,<br>3 = widow/er, 4 = divorced, partnership dissolved |
| nationality | Not swiss (1) / Swiss (2) |
| secondary residence | no (1) / yes (2) |
| Household Size | 1, 2, [3,5], [6,10] >10 |
| Income | unknown (In OASI = no), $(0,12000]$, $(12000,24000]$, $(24000,48000]$,<br>$(48000,72000]$, $(72000,96000]$, $(96000,120000]$, $> 120000$ |
| OASItri | 1 = in OASI only Jan-March, 0 = otherwise |

# 2 Considered estimators

Let $U$ be the target population of size $N$; in this project, $U$ is the set of individuals in the STATPOP data set. This population is composed of $D$ non-overlapping areas $U_1,\ldots,U_D$; in this case, the Swiss districts, of sizes $N_1,\ldots,N_D$ with $N=\sum_{d=1}^{D} N_d$. Let $s$ be a sample of size $n$ drawn from $U$; in this case, $s$ is the set of individuals in the Structural Survey. Let $s_d$ the subsample from area (or district) $d$ of size $n_d$, $d=1,\ldots,D$, where $n=\sum_{d=1}^{D} n_d$. Let $\bar{s}_d=U_d-s_d$ denote the complement of the sample from area $d$. Let $Y_{di}\in\{0,1\}$ be the target variable for unit $i$ in area $d$; here, $Y_{di}=1$ stands for "active" and $Y_{di}=0$ for "non-active". The target parameters are the area proportions

$$P_d=N_d^{-1}\sum_{i=1}^{N_d} Y_{di},\quad d=1,\ldots,D.$$

If $w_{di}$ is the calibrated sampling weight of $i$-th unit within $d$-th area, the GREG estimator is given by

$$\hat{P}_d^{GREG}=\frac{1}{\hat{N}_d}\sum_{i\in s_d} w_{di}Y_{di},$$

where $\hat{N}_d=\sum_{i\in s_d} w_{di}$. This estimator is (practically) design unbiased; however, it is inefficient for areas with small sample sizes because it uses only the area-specific sample data.

Phase I has shown that the empirical best linear unbiased predictors (EBLUPs) based on a LMM with the selected sets of covariates perform significantly better than the GREG estimators in terms of total MSE, both under the model and the design approaches, for practically all districts. The LMM assumes that the population variables $Y_{di}$ satisfy a linear regression model including random district effects representing the unexplained between-area variability. More specifically, it assumes that

$$\begin{aligned}&Y_{di}=\mathbf{x}_{di}'\boldsymbol{\beta}+u_d+e_{di},\\&u_d\overset{iid}{\sim}N(0,\sigma_u^2),\quad e_{di}\overset{iid}{\sim}N(0,\sigma_e^2),\quad i=1,\ldots,N_d,\quad d=1,\ldots,D,\end{aligned}\tag{1}$$

where $u_d$ is the random effect for district $d$. Although normality is specified in (1), the best linear unbiased predictor (BLUP) derived from this model does not require normality. Moreover, even if normality does not hold, maximum likelihood (ML) and restricted ML (REML) estimates of the model parameters obtained from the normal likelihood are still consistent under regularity assumptions (Jiang 1996). In fact, we have seen in Phase I of the project that a LMM provides practically the same small area estimates as a logistic GLMM with the same set of covariates due to the fact that the true proportions of active people are in the interval (0.2,0.8), in which the logit function is approximately linear.

For details on the ML fitting of mixed models, see Hartley and Rao (1967). Here we focus on REML estimates (Patterson and Thompson 1971; 1974), which have smaller bias for finite sample size. Let $\hat{\boldsymbol{\beta}}$ be the weighted least squared estimator of $\boldsymbol{\beta}$ and $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$ be the restricted ML (REML) estimators of $\sigma_u^2$ and $\sigma_e^2$ based on the normal likelihood. The EBLUP of $P_d$ under this model is given by

$$\hat{P}_d^{EBLUP}=\frac{1}{N_d}\left(\sum_{i\in s_d} Y_{di}+\sum_{i\in\bar{s}_d}\hat{Y}_{di}\right),\quad d=1,\ldots,D,\tag{2}$$

where $\hat{Y}_{di} = \mathbf{x}'_{di}\hat{\boldsymbol{\beta}} + \hat{u}_d$ is the predicted value of $Y_{di}$ obtained by fitting the model. Here, $\hat{u}_d = \hat{\gamma}_d(\bar{y}_d - \bar{\mathbf{x}}'_d\hat{\boldsymbol{\beta}})$ is the BLUP of $u_d$, where $\hat{\gamma}_d = \hat{\sigma}_u^2/(\hat{\sigma}_u^2 + \hat{\sigma}_e^2/n_d)$, $\bar{y}_d = n_d^{-1}\sum_{i\in s_d} Y_{di}$ and $\bar{\mathbf{x}}_d = n_d^{-1}\sum_{i\in s_d}\mathbf{x}_{di}$.

A desirable property of small area estimators is that the estimated totals for the areas add up to a reliable estimator of the population total. This property is called the benchmarking property. A reliable estimator of the population total $Y = \sum_{d=1}^{D}\sum_{i=1}^{N_d} Y_{di}$ is the GREG estimator

$$\hat{Y}^{GREG} = \sum_{d=1}^{D}\sum_{i\in s_d} w_{di}Y_{di} = \sum_{d=1}^{D}\hat{N}_d\hat{P}_d^{GREG}.$$

A simple adjustment of the EBLUP based on the LMM to make it satisfy the benchmarking property is the ratio-adjustment

$$\hat{P}_d^{BM} = \hat{P}_d^{EBLUP}\frac{\hat{Y}^{GREG}}{\sum_{d=1}^{D} N_d\hat{P}_d^{EBLUP}}, \quad d = 1,\ldots,D. \tag{3}$$

This estimator is called hereafter benchmarked EBLUP.

# 3 Parametric bootstrap estimator of the model MSE

Analytical approximations to the model MSE of the EBLUP are obtained in the literature only when normality holds and for the number of areas $D$ tending to infinity. In our problem, target variables $Y_{di}$ are binary and therefore the available analytical approximations are not valid. Moreover, even if an analytical formula was available for the estimated MSE of the unadjusted EBLUP, this MSE estimator is not necessarily good for the benchmarked estimator. Note that the adjustment factor for the EBLUP given in (3) is random and therefore analytical approximation of the MSE of the benchmarked estimator is not straightforward. Thus, here we appeal to a parametric bootstrap procedure that can handle complex estimators similarly as in the case of simple estimators. The selected parametric bootstrap procedure is especially designed for finite populations and was first introduced by González-Manteiga *et al.* (2008). It follows the steps below:

1) Fit the LMM model (1) to the available sample data $\{(\mathbf{x}_{di}, Y_{di}); i \in s_d,\ d = 1,\ldots,D\}$, obtaining model parameter estimates $\hat{\boldsymbol{\beta}}$, $\hat{\sigma}_u^2$ and $\hat{\sigma}_e^2$.

2) Generate bootstrap random effects as $u_d^* \stackrel{iid}{\sim} N(0,\hat{\sigma}_u^2)$, $d = 1,\ldots,D$.

3) Generate bootstrap population values as

$$Y_{di}^* \stackrel{ind.}{\sim} N(\mathbf{x}'_{di}\hat{\boldsymbol{\beta}} + u_d^*, \hat{\sigma}_e^2), \quad i = 1,\ldots,N_d,\ d = 1,\ldots,D.$$

Although normality does not really hold because $Y_{di}$ are binary, a bootstrap procedure in which the bootstrap population values $Y_{di}^*$ are generated from a logistic GLMM was also implemented and the simulation results were practically identical to those obtained from this bootstrap procedure.

4) Calculate the true bootstrap proportions of interest

$$P_d^* = \frac{1}{N_d}\sum_{i=1}^{N_d} Y_{di}^*, \quad d = 1,\ldots,D.$$

5) Select the part of the bootstrap population corresponding to the sample units, called bootstrap sample data: $\{(\mathbf{x}_{di}, Y_{di}^*); i \in s_d,\ d = 1,\ldots,D\}$. Now fit the LMM model (1) to the bootstrap sample data, obtaining bootstrap model parameter estimates $\hat{\boldsymbol{\beta}}^*$, $\hat{\sigma}_u^{2*}$, $\hat{\sigma}_e^{2*}$, and predicted random effects $\hat{u}_d^*$, $d = 1,\ldots,D$. Calculate the EBLUPs $\hat{P}_d^{EBLUP*}$ using the bootstrap sample data, as

$$\hat{P}_d^{EBLUP*} = \frac{1}{N_d}\left(\sum_{i\in s_d} Y_{di}^* + \sum_{i\in \bar{s}_d} \hat{Y}_{di}^*\right), \quad d = 1,\ldots,D,$$

where $\hat{Y}_{di}^* = \mathbf{x}_{di}'\hat{\boldsymbol{\beta}}^* + \hat{u}_d^*$ is the predicted value of $Y_{di}^*$ obtained by fitting the LMM to the bootstrap sample data. Calculate also the benchmarked EBLUPs $\hat{P}_d^{BM*}$ as

$$\hat{P}_d^{BM*} = \hat{P}_d^{EBLUP*}\frac{\hat{Y}^{GREG}}{\sum_{d=1}^{D} N_d \hat{P}_d^{EBLUP*}},$$

6) Repeat Steps 2–5 for $b = 1,\ldots,B$, where $B$ is large. Let $P_d^{*(b)}$ be the true proportion, $\hat{P}_d^{EBLUP*(b)}$ be the EBLUP and $\hat{P}_d^{BM*(b)}$ be the benchmarked EBLUP obtained in $b$-th bootstrap replicate. The parametric bootstrap estimator of the model MSE of the EBLUP, $\hat{P}_d^{EBLUP}$, is given by

$$\text{mse}_{PB}(\hat{P}_d^{EBLUP}) = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{P}_d^{EBLUP*(b)} - P_d^{*(b)}\right)^2. \tag{4}$$

Similarly, for the benchmarked EBLUP $\hat{P}_d^{BM}$, the parametric bootstrap MSE estimator is given by

$$\text{mse}_{PB}(\hat{P}_d^{BM}) = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{P}_d^{BM*(b)} - P_d^{*(b)}\right)^2. \tag{5}$$

# 4 Nonparametric bootstrap estimator of the design MSE

The design MSE is obtained by averaging the squared errors over the possible samples drawn from a fixed population using the considered sampling design. Here we propose a nonparametric bootstrap for the estimation of this design MSE. This procedure follows the steps below:

1) Replicate each data point $(\mathbf{x}_{di}, Y_{di})$ from the sample a number of times equal to the rounded calibrated sampling weight $w_{di}$. This leads to the bootstrap population data set $\{(\mathbf{x}_{di}^*, Y_{di}^*); i = 1,\ldots,\hat{N}_d,\ d = 1,\ldots,D\}$.

2) Calculate the true bootstrap proportions of interest

$$P_d^* = \frac{1}{\hat{N}_d} \sum_{i=1}^{\hat{N}_d} Y_{di}^*, \quad d = 1, \ldots, D.$$

3) Draw a simple random sample $s_d^*$ from each district $d$. Select the corresponding bootstrap elements for that sample: $\{(\mathbf{x}_{di}^*, Y_{di}^*); i \in s_d^*,\ d = 1, \ldots, D\}$. Now fit the LMM model (1) to these bootstrap sample data, obtaining bootstrap model parameter estimates $\hat{\boldsymbol{\beta}}^*$, $\hat{\sigma}_u^{2*}$, $\hat{\sigma}_e^{2*}$, and predicted random effects $\hat{u}_d^*$, $d = 1, \ldots, D$. Calculate the EBLUPs $\hat{P}_d^{EBLUP*}$ using the bootstrap sample data,

$$\hat{P}_d^{EBLUP*} = \frac{1}{N_d} \left( \sum_{i \in s_d^*} Y_{di}^* + \sum_{i \in \bar{s}_d^*} \hat{Y}_{di}^* \right), \quad d = 1, \ldots, D,$$

where $\hat{Y}_{di}^* = \mathbf{x}_{di}^{*\prime} \hat{\boldsymbol{\beta}}^* + \hat{u}_d^*$ is the predicted value of $Y_{di}^*$ obtained by fitting the LMM. Calculate also the benchmarked EBLUPs $\hat{P}_d^{BM*}$ as

$$\hat{P}_d^{BM*} = \hat{P}_d^{EBLUP*} \frac{\hat{Y}^{GREG}}{\sum_{d=1}^{D} \hat{N}_d \hat{P}_d^{EBLUP*}},$$

4) Repeat Step 3 for $b = 1, \ldots, B$, where $B$ is large. Note that here the true bootstrap proportions $P_d^*$ are constant over bootstrap replicates because the bootstrap population in Step 1 is fixed. Let $\hat{P}_d^{EBLUP*(b)}$ be the EBLUP and $\hat{P}_d^{BM*(b)}$ be the benchmarked EBLUP obtained in $b$-th bootstrap replicate. The nonparametric bootstrap estimator of the design MSE of $\hat{P}_d^{EBLUP}$ is given by

$$\mathrm{mse}_{NPB}(\hat{P}_d^{EBLUP}) = \frac{1}{B} \sum_{b=1}^{B} \left( \hat{P}_d^{EBLUP*(b)} - P_d^* \right)^2.$$

Similarly, for the benchmarked EBLUP $\hat{P}_d^{BM}$, the nonparametric bootstrap estimator of the design MSE is given by

$$\mathrm{mse}_{NPB}(\hat{P}_d^{BM}) = \frac{1}{B} \sum_{b=1}^{B} \left( \hat{P}_d^{BM*(b)} - P_d^* \right)^2. \tag{6}$$

In contrast with the parametric bootstrap of Section 3, which generates new population data in each bootstrap replicate, note that this nonparametric bootstrap procedure is based only on the original sample data, which is simply replicated using the sampling weights. In fact, the average over the bootstrap replicates in (6) actually estimates the average over the possible subsamples $s_d$ from district $d$, which are all based on the same set of $n_d$ units. Thus, the nonparametric bootstrap MSE estimate (6) for district $d$ might be inefficient for a district with small sample size $n_d$.

# 5 Simulation studies for the parametric bootstrap MSE

A model-based simulation study was performed to analyze the performance of (4) and (5) as estimators of the corresponding model MSEs of the EBLUP and the benchmarked EBLUP. Since the target variable is actually binary, in this simulation study we consider that the population data are generated by the GLMM

$$\begin{aligned} &Y_{di}|v_d \sim \text{Bern}(p_{di}),\ \log\left(\frac{p_{di}}{1-p_{di}}\right) = \mathbf{x}_{di}'\boldsymbol{\alpha} + v_d \\ &v_d \stackrel{iid}{\sim} N(0,\sigma_v^2). \end{aligned} \tag{7}$$

In this way, the true MSE in this simulation study will incorporate any potential bias due to considering a LMM instead of a GLMM in the parametric bootstrap procedure.

To make the simulation study realistic, we consider as true values of the model parameters $\boldsymbol{\alpha}$ and $\sigma_v^2$ in (7), those obtained by fitting the model (7) to the data from the Structural Survey. Thus, using those fitted values as the true model parameters, to approximate the true model MSE with good precision, we generate $L = 1,000$ Monte Carlo populations from model (7). Let $P_d^{(\ell)}$ be the true proportion for $d$-th area in $\ell$-th Monte Carlo population. From each generated population, we draw a stratified sample with districts acting as strata and simple random sampling (SRS) within each district. The district sample sizes were taken as in the simulation studies in Phase I of the project, namely $n_d = 60 + 5(k-1)$, $k = 1,\dots,D$ for $D = 141$. The sample units are kept fixed over the $L$ Monte Carlo replicates because it is a purely model-based simulation study. Let $\hat{P}_d^{EBLUP(\ell)}$ and $\hat{P}_d^{BM(\ell)}$ be the EBLUP and benchmarked EBLUP obtained using the sample data from $\ell$-th population. For the benchmarked EBLUPs, the true model MSEs were approximated as

$$\text{MSE}_m(\hat{P}_d^{BM}) = L^{-1}\sum_{\ell=1}^{L}(\hat{P}_d^{BM(\ell)} - P_d^{(\ell)})^2, \quad d = 1,\dots,D.$$

The true model MSEs of the unadjusted EBLUPs for each district were approximated similarly but replacing $\hat{P}_d^{BM(\ell)}$ by $\hat{P}_d^{EBLUP(\ell)}$.

Now to approximate the expected value of the parametric bootstrap MSE estimates, the simulation study was repeated generating $L = 100$ Monte Carlo populations. With the sample data from $\ell$-th generated population, we carried out the parametric bootstrap procedure of Section 3 with number of bootstrap replicates $B = 100$, obtaining parametric bootstrap MSE estimates for the benchmarked estimators denoted by $\text{mse}_{PB}^{(\ell)}(\hat{P}_d^{BM})$, $\ell = 1,\dots,L$. Then, the expected value of the MSE estimates is approximated empirically by averaging the bootstrap estimates over Monte Carlo replicates as

$$L^{-1}\sum_{\ell=1}^{L}\text{mse}_{PB}^{(\ell)}(\hat{P}_d^{BM}), \quad d = 1,\dots,D.$$

Similarly it was done for the unadjusted EBLUPs.

Results for the benchmarked EBLUP are depicted in Figure 1 for each district in the $x$ axis, with districts sorted by decreasing sample sizes (labels in the $x$ axis are sample sizes).

Figure 1 shows that the parametric bootstrap estimates (labeled BOOTSTRAP) are tracking rather well the empirical values of the true model MSE (labeled TRUE), as expected from a bootstrap procedure. In fact, the slight more variability of the empirical approximations to the true model MSEs is very probably due to simulation noise and it would get reduced by increasing the number of Monte Carlo simulations let us say to $L = 10,000$. For the unadjusted EBLUPs, the parametric bootstrap procedure performs very similarly.

It is also interesting to analyze whether the parametric bootstrap MSE, which is an estimator of the model MSE, $\mathrm{MSE}_m(\hat{P}_d^{BM})$, is also an acceptable estimator of the design MSE, $\mathrm{MSE}_\pi(\hat{P}_d^{BM})$. For this purpose, a new simulation study was carried out under the design-based setup, considering the Structural Survey data as the (fixed) true population and drawing samples out of it. To approximate empirically the true design MSE, a first simulation study was performed drawing $L = 10,000$ samples out of the population. The district sample sizes were taken as in the model-based simulation study described above. Let $P_d$ be the true proportion for district $d$, and $\hat{P}_d^{EBLUP(\ell)}$ and $\hat{P}_d^{BM(\ell)}$ be the estimates obtained using the data from $\ell$-th sample. The true design MSEs of the benchmarked EBLUPs were approximated as

$$\mathrm{MSE}_\pi(\hat{P}_d^{BM}) = L^{-1}\sum_{\ell=1}^{L}(\hat{P}_d^{BM(\ell)} - P_d)^2, \quad d = 1,\dots,D.$$

The true design MSEs of the unadjusted EBLUPs $\hat{P}_d^{BM}$ for each district $d$ were approximated similarly.

Now to estimate the expected value of the parametric bootstrap MSE estimates under the design-based approach, the simulation study was repeated drawing now $L = 100$ samples from the given population. With the data from $\ell$-th sample, we carried out the parametric bootstrap procedure with number of bootstrap replicates $B = 100$, obtaining parametric bootstrap MSE estimates of the benchmarked estimates $\mathrm{mse}_{PB}^{(\ell)}(\hat{P}_d^{BM})$, $\ell = 1,\dots,L$. Then, we averaged these bootstrap estimates over Monte Carlo samples as

$$L^{-1}\sum_{\ell=1}^{L}\mathrm{mse}_{PB}^{(\ell)}(\hat{P}_d^{BM}), \quad d = 1,\dots,D.$$

The analogous formula is applied for the unadjusted EBLUPs. We have already mentioned that the parametric bootstrap procedure estimates the model MSE and not the design MSE. However, if the model is approximately correct, the average of these MSEs over a large number of areas should be similar. Thus, the parametric bootstrap estimate is expected to estimate correctly the design MSE in average but not in each particular area. Figure 2 plots the true design MSE together with the parametric bootstrap MSE estimates for each district, with districts sorted by decreasing sample sizes. This figure shows that the parametric bootstrap model MSE estimates track surprisingly well the design MSEs for all districts except for three districts with smaller sample sizes, in which the parametric bootstrap seems to underestimate the design MSE. However, note that in these simulations, the true sample sizes in the Structural Survey are acting as population sizes. Then, for those districts, the population sizes are actually small and therefore the true design MSEs are somewhat unstable for those areas. This instability is not likely to happen in the true data because the smallest district population size is very large, $\min(N_d) = 1839$.

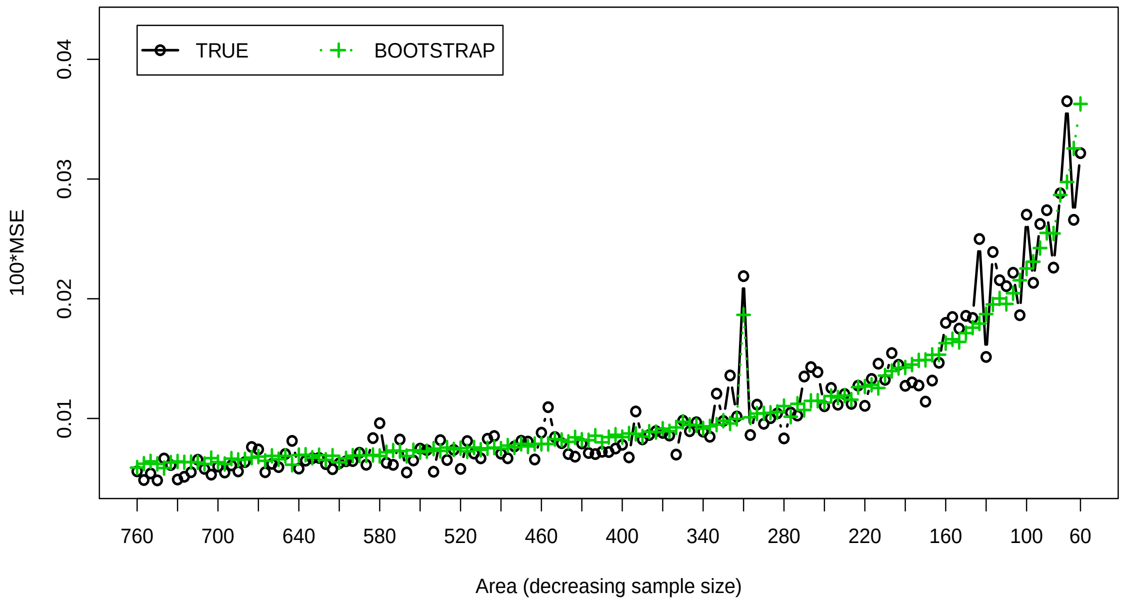


Figure 1: True model MSE (labeled TRUE) of the benchmarked EBLUP based on the LMM and parametric bootstrap estimator (labeled BOOTSTRAP). Districts sorted by decreasing sample sizes.

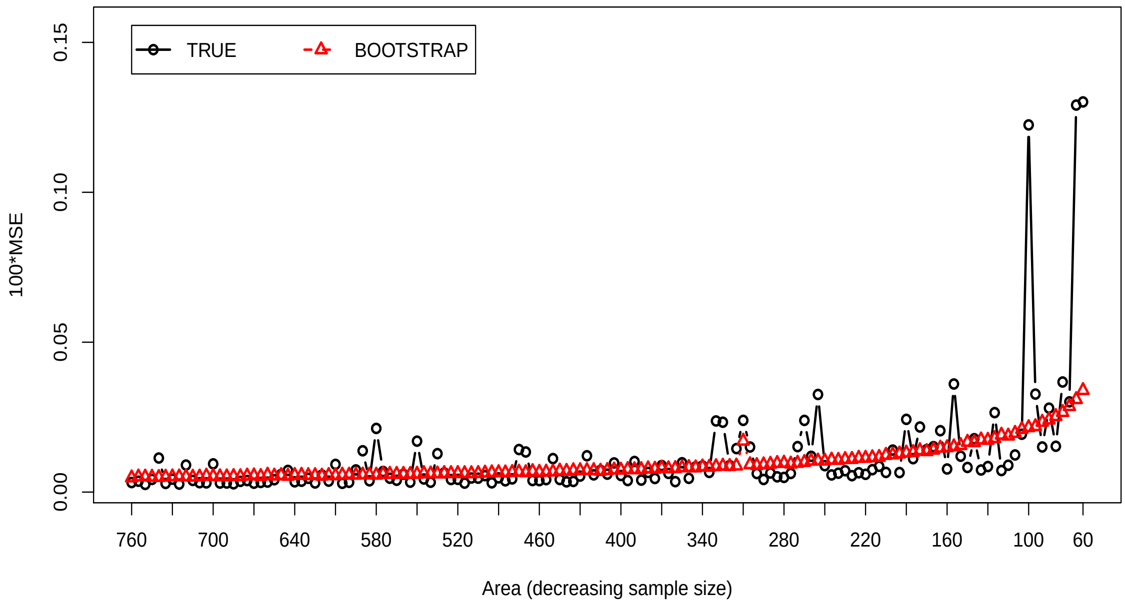


Figure 2: True design MSE (labeled True) of the benchmarked EBLUP based on the LMM and parametric bootstrap estimator (labeled Bootstrap). Districts sorted by decreasing sample sizes.

# 6 Results for the STATPOP data set

Table 2 in Appendix 3 gives, for each district within each stratum, the district sample sizes, the different estimates of the percentages of active people (GREG, EBLUPs based on the LMM and benchmarked EBLUPs) with their estimated percent RRMSEs obtained using the parametric bootstrap procedure. These results can be better analyzed with the aid of figures. Figure 3 plots the EBLUPs based on the LMM of the district percentages of active people (labeled LMM), together with the benchmarked EBLUPs (labeled LMM (BM)) for each district. See that the two sets of estimates are very close to each other, with the benchmarked estimates just slightly larger than the unadjusted EBLUPs. In fact, the benchmarking adjustment turned out to be 1.0076, which means a very mild adjustment.

Figure 4 represents a line plot of the benchmarked EBLUPs against the GREG estimates. Since the GREG estimators are approximately design-unbiased, a cloud of points all above or below the line $y = x$ would suggest a systematic design bias of the EBLUPs. This does not seem to be the case because the points turn out to be around the line $y = x$, with points distributed at both sides. The group of points that appear below the line close to the top right corner indicate some deviation of the EBLUPs to the GREGs for those districts. But note that these points correspond to large GREG estimates of the proportions of active people. The points are a little further apart from the line because, according to the model, which is supposed to fit well the data, these districts should not have such large proportions of active people. Take into account that GREG estimates tend to vary more than they should due to the small district sample sizes. Thus, the model is smoothing those more extreme proportions and providing more reasonable estimates according to the model. In contrast, the points with large GREG estimates that appear close to the line correspond to districts in which the extreme GREG estimated proportions are explained by the considered auxiliary variables in the LMM model.

Figure 5 gives a different display of the two sets of estimates, for each district in the $x$ axis. We can see that the estimates are practically the same for the large districts (on the left-hand side of the plot), but for the districts with smaller sample sizes (on the right-hand side), the two estimates present slight deviations. We know that for districts with small sample sizes, the GREG estimators can be inefficient as shown in Phase I of the project. Thus, here we consider the benchmarked EBLUPs as more reliable estimates.

Figure 6 plots the percent relative root MSE (RRMSE) estimates obtained by the proposed parametric bootstrap procedure with $B = 250$ replicates, for the unadjusted EBLUPs based on the LMM (labeled LMM) and the benchmarked EBLUPs (labeled LMM (BM)). See that the estimated RRMSE is about 0.5% larger for the benchmarked estimates in the districts with largest sample sizes, but the difference decreases with the district sample size. Still, 0.5% is not a stricking RRMSE increase. The decrease of the differences when decreasing the district sample size seems to be an artifact of estimating the RRMSE which is a ratio of the root MSE over the estimate. This decrease of the differences does not appear when looking at the (non-relative) estimated MSEs, see Figure 7. Concerning computation time, for the STATPOP data, the parametric bootstrap procedure with $B = 250$ replicates takes less than 22 hours in a 3.40-3.90 gHz PC with an Intel Core i7 processor.

Figure 8 plots the estimated model MSE using the parametric bootstrap method together with the estimated design MSE using the nonparametric bootstrap procedure, for the benchmarked EBLUPs based on the LMM. Note that these two estimates have a different target parameter, which is the model MSE in the former and the design MSE in the latter. Thus, in principle they do not need to agree. However, if they were good estimates of their corresponding true MSEs and the model was correct, they should show a similar pattern because they will be similar when averaging across a large number of districts, see Theorem 1 in Appendix 1. By the simulation studies performed in Section 5, we know that the parametric bootstrap procedure estimates correctly the model MSE for all districts and it also tracks acceptably well the design MSE for the districts with not so small sample sizes. In contrast, we do not have information on how the nonparametric bootstrap tracks its corresponding design MSE. Still, we can see that both procedures seem to agree for districts with sample sizes above $n_d = 350$, but the nonparametric bootstrap estimates of the design MSE give very large MSEs for districts with smaller sample sizes. We do not really know which estimate performs better for the design MSE, but the very large MSE estimates for the districts with smaller sample sizes seems to suggest some instability of the nonparametric bootstrap estimate of the design MSE. We would recommend the use of the nonparametric bootstrap MSE estimates only if one wishes to interpret the given uncertainty measures under the design-based approach and is strongly averse to potential MSE underestimation.

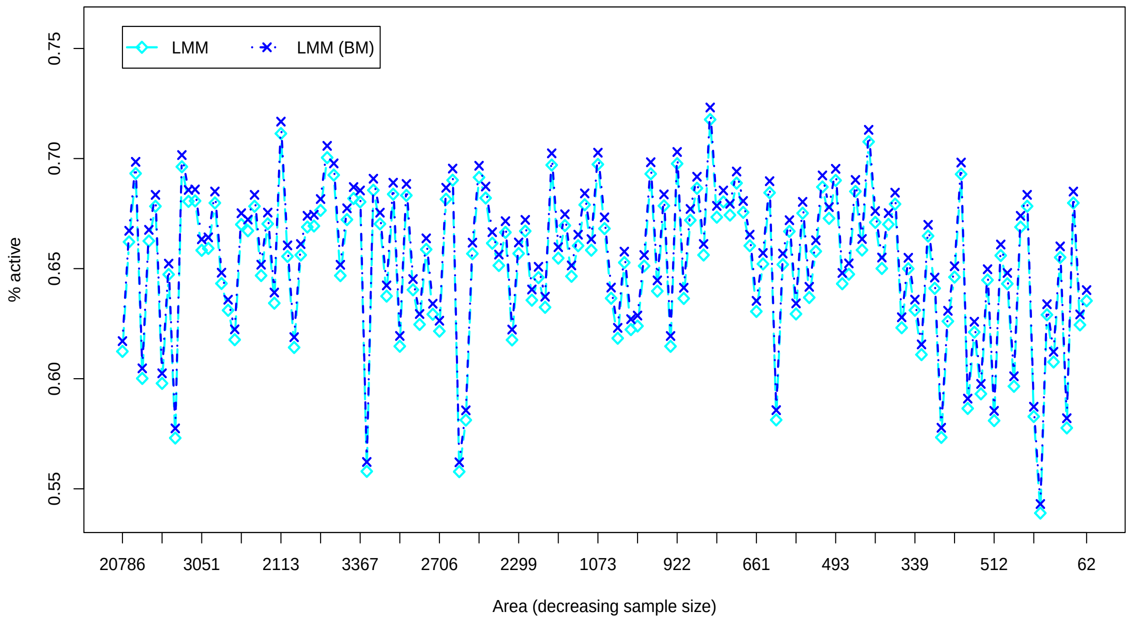


Figure 3: Estimated district percentages of active people using the EBLUPs and the benchmarked EBLUPs based on the LMM, with districts sorted by decreasing sample size.

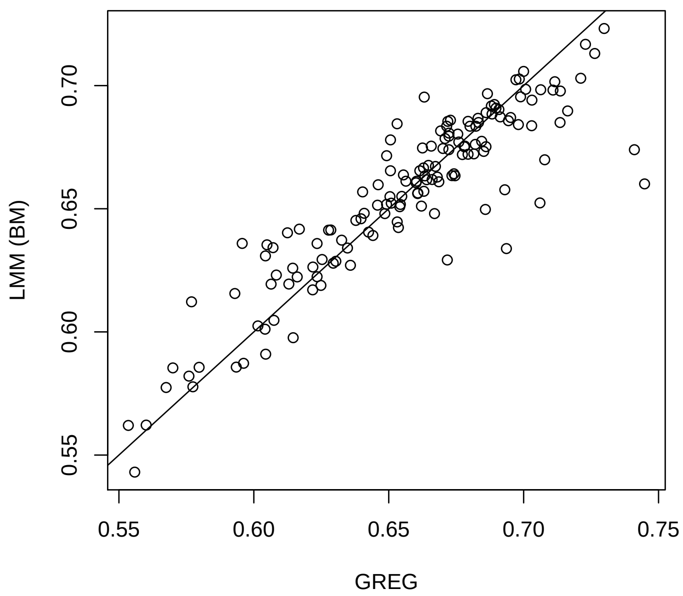


Figure 4: Estimated district percentages of actives using the benchmarked EBLUPs based on the LMM model against GREG estimates.

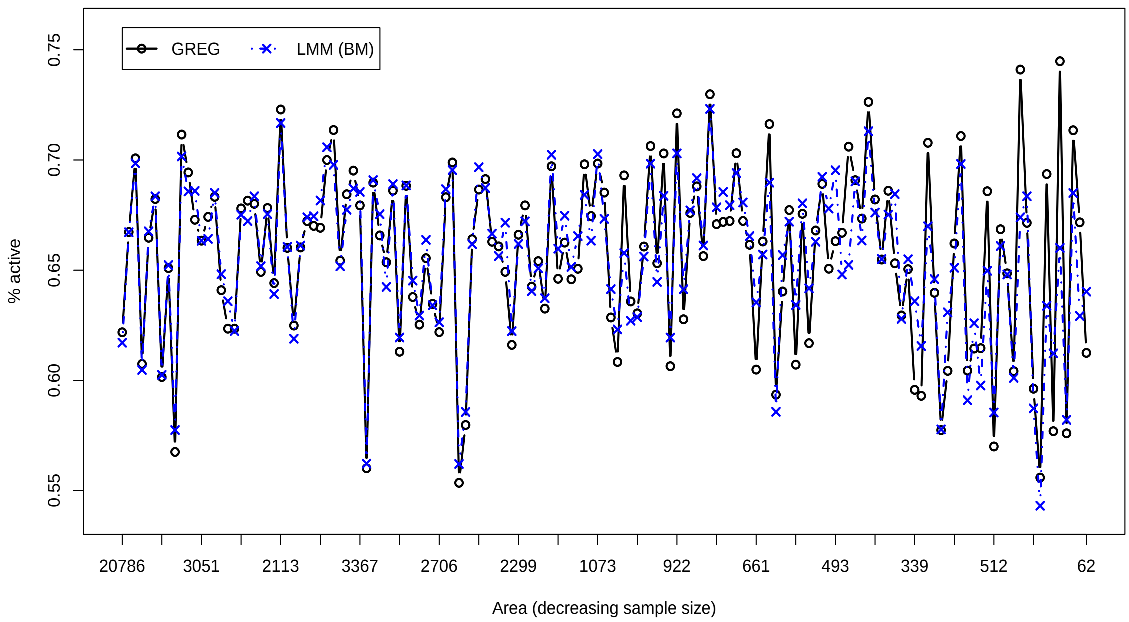


Figure 5: Estimated district percentages of actives using the GREG estimator and the benchmarked EBLUPs based on the LMM, with districts sorted by decreasing sample size.

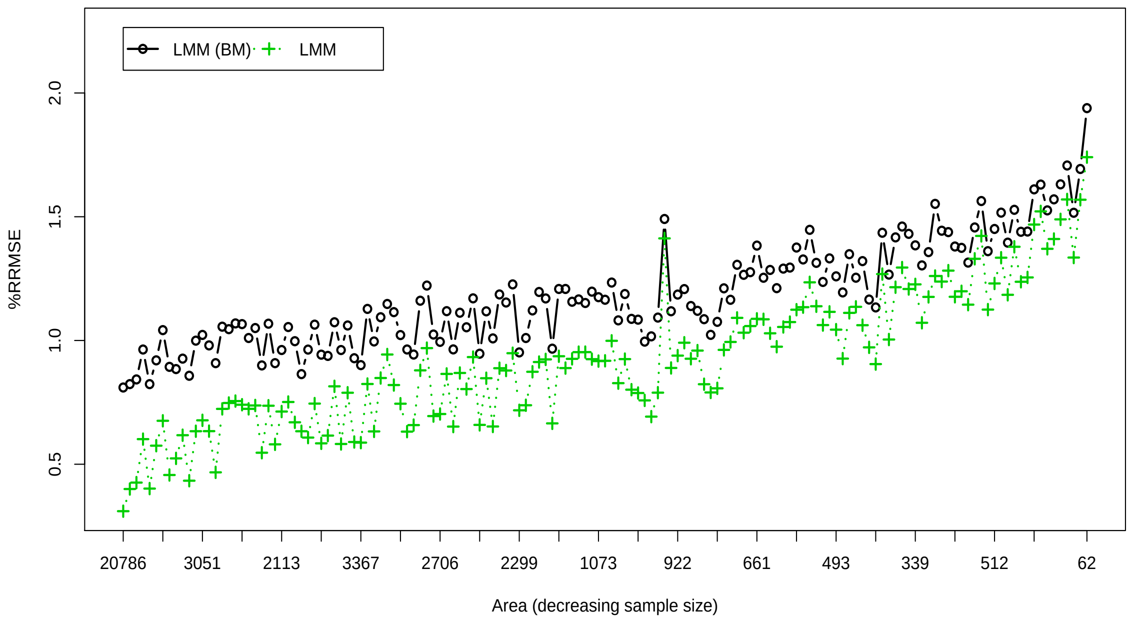


Figure 6: Estimated percent RRMSEs obtained by parametric bootstrap for the unadjusted EBLUPs (labeled LMM) and the benchmarked EBLUPs (labeled LMM(BM)) for each district, with districts sorted by decreasing sample size.

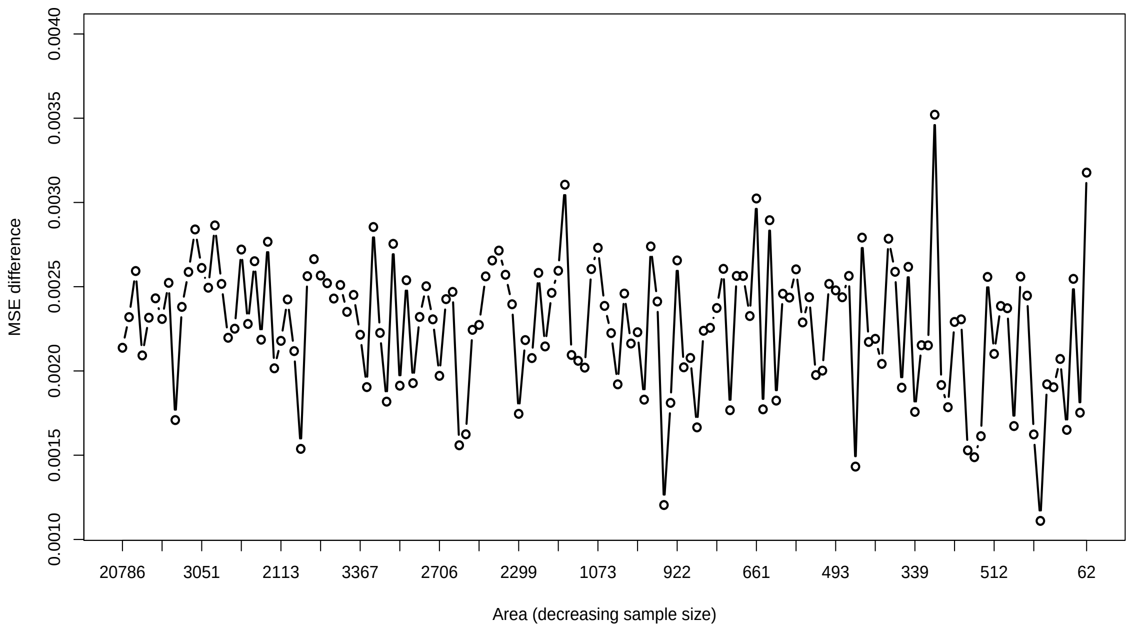


Figure 7: Difference between parametric bootstrap MSE estimates for the benchmarked EBLUPs and the unadjusted EBLUPs for each district, with districts sorted by decreasing sample size.

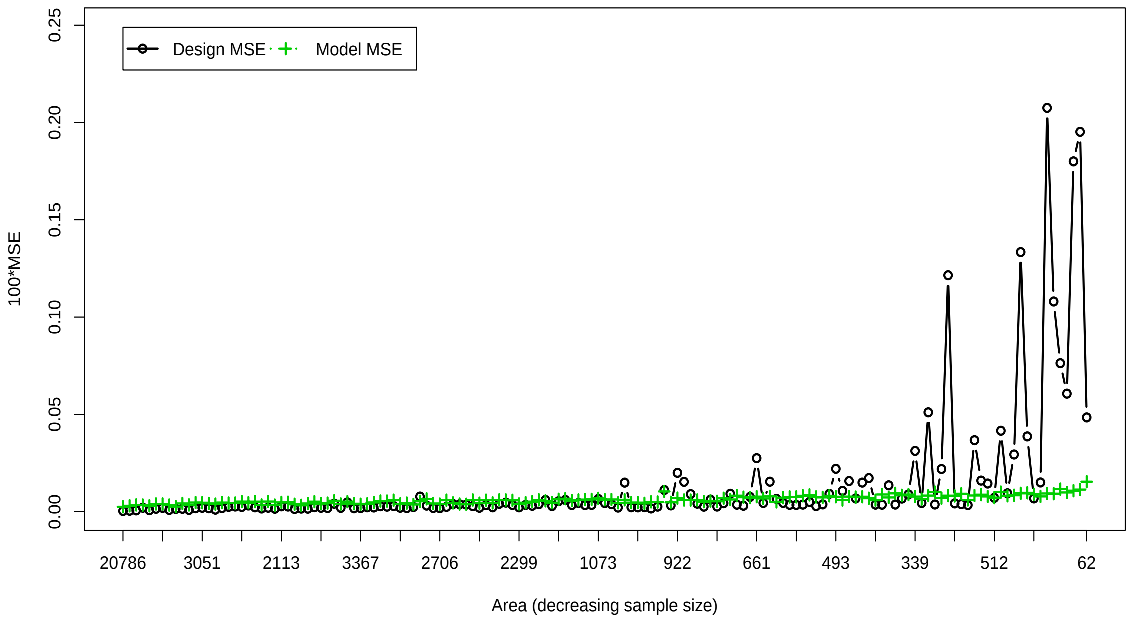


Figure 8: Estimated model MSEs using the parametric bootstrap (labeled "Model MSE") together with estimated design MSE using the nonparametric bootstrap (labeled "Design MSE"), for the benchmarked EBLUPs based on the LMM, with districts sorted by decreasing sample size.

# 7 Concluding remarks

Below we summarize the main achievements of the two phases of this project:

- A rich and powerful regression model has been found for the activity in Switzerland. This has an important economic value itself, since the model might help to understand the factors explaining the activity, and this might provide relevant information for the design of specific social policies or programs related with the labor force.

- Efficient estimators of the proportions of active people in the Swiss districts have been found. The selected model explains a large part of the between-district variability in the activity and therefore provides estimates of better quality than the current GREG estimates. The design-based simulation with true data carried out in Phase I of the project showed that the estimates (EBLUPs) obtained from the selected model (LMM) achieve a significant reduction in relative error in comparison with the GREG estimates for practically all districts, see Figure 57 of the report for Phase I. Indeed, when averaging over districts of similar sample sizes (Figure 59 of the Phase I report), the (true) design RRMSE of the benchmarked EBLUPs based on the LMM is less than half of the RRMSE of the GREG estimates even for districts with large sample sizes.

Thus, for estimation at the district level, large RRMSE reductions are achieved without increasing the district survey sample sizes. This is achieved thanks to a clever use of the available auxiliary information to establish relationships among all the districts, which helps to borrow strength from all districts when estimating in a particular one.

- Bootstrap procedures have been proposed for estimating both model MSE and design MSE. Estimates of design MSEs are typically unstable because they are based on the district-specific sample size. As shown in the simulation studies of Section 5, the parametric bootstrap procedure estimates correctly the corresponding model MSE and it also gives acceptable estimates of the design MSE for the districts with sample sizes over $n_d = 350$. The nonparametric bootstrap procedure for estimating the design MSE seems to be also good for the districts with larger sample sizes ($n_d \geq 350$) but might be unstable for districts with smaller sample sizes ($n_d < 350$).

- Finally, using the whole STATPOP data and the Structural Survey data, we have computed the EBLUPs together with their benchmarked counterparts, for which the estimated district totals add up to the GREG estimate of the population total. The benchmarking adjustment turns out to be very mild in the true data, although this mild adjustment still leads to a small increase in RRMSE. Still, the estimated RRMSEs of the benchmarked estimates remain below 2% even for the smallest districts, see Figure 6. Thus, the benchmarked EBLUPs represent more efficient alternatives to the current GREG district estimates, and their MSEs can be estimated using one of the parametric bootstrap methods described in Section 3.

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# Appendix 1: Design versus model mean squared error

This appendix relates the model MSE and the design MSE of the EBLUP (2) under the nested error model (1) with known model parameters and with SRS within each area, when domain population sizes $N_d$ are large and area sampling fractions $f_d = n_d/N_d$ are negligible. Note that when model parameters are known, the EBLUP equals the BLUP, and for $f_d = n_d/N_d \approx 0$, the BLUP can be expressed as a convex linear combination of the survey regression estimator $\tilde{P}_d^{SR} = \bar{y}_d + (\bar{\mathbf{X}}_d - \bar{\mathbf{x}}_d)'\boldsymbol{\beta}$ and the regression-synthetic estimator $\bar{\mathbf{X}}_d'\boldsymbol{\beta}$ as follows

$$\tilde{P}_d^{BLUP} = \gamma_d \tilde{P}_d^{SR} + (1-\gamma_d)\bar{\mathbf{X}}_d'\boldsymbol{\beta}, \tag{8}$$

where $\gamma_d = \sigma_u^2/(\sigma_u^2 + \sigma_e^2/n_d)$ and $\bar{\mathbf{X}}_d = N_d^{-1}\sum_{i=1}^{N_d}\mathbf{x}_{di}$. The model MSE of the BLUP with known $\boldsymbol{\beta}$ is given by

$$\text{MSE}_m(\tilde{P}_d^{BLUP}) = \gamma_d \frac{\sigma_e^2}{n_d}. \tag{9}$$

Regarding the design MSE, first note that under SRS without replacement, the survey regression estimator is design-unbiased, because

$$E_\pi(\tilde{P}_d^{SR}) = E_\pi(\bar{y}_d) + \left\{\bar{\mathbf{X}}_d - E_\pi(\bar{\mathbf{x}}_d)\right\}'\boldsymbol{\beta} = P_d + (\bar{\mathbf{X}}_d - \bar{\mathbf{X}}_d)'\boldsymbol{\beta} = 0.$$

Moreover, since under the design, $\bar{\mathbf{X}}_d$ is fixed, its sampling variance is given by

$$V_\pi(\tilde{P}_d^{SR}) = V_\pi\left\{\bar{y}_d + (\bar{\mathbf{X}}_d - \bar{\mathbf{x}}_d)'\boldsymbol{\beta}\right\} = V_\pi\left(\bar{y}_d - \bar{\mathbf{x}}_d'\boldsymbol{\beta}\right).$$

Defining $\varepsilon_{di} = Y_{di} - \mathbf{x}_{di}'\boldsymbol{\beta} = u_d + e_{di}$ and $\bar{\varepsilon}_d = n_d^{-1}\sum_{i\in s_d}\varepsilon_{di}$, we have

$$V_\pi(\tilde{P}_d^{SR}) = V_\pi(\bar{\varepsilon}_d) = (1-f_d)\frac{S_{\varepsilon d}^2}{n_d},$$

where $S_{\varepsilon d}^2 = (N_d-1)^{-1}\sum_{i=1}^{N_d}(\varepsilon_{di} - \bar{E}_d)^2$ and $\bar{E}_d = N_d^{-1}\sum_{i=1}^{N_d}\varepsilon_{di}$.

Now, the design MSE of the BLUP given in (8) is

$$\begin{aligned}
\text{MSE}_\pi(\tilde{P}_d^{BLUP}) &= E_\pi(\tilde{P}_d^{BLUP} - P_d)^2 \\
&= E_\pi\left\{\gamma_d(\tilde{P}_d^{SR} - P_d) + (1-\gamma_d)(\bar{\mathbf{X}}_d'\boldsymbol{\beta} - P_d)\right\}^2 \\
&= \gamma_d^2 V_\pi(\tilde{P}_d^{SR}) + (1-\gamma_d)^2(\bar{\mathbf{X}}_d'\boldsymbol{\beta} - P_d)^2 \\
&= \gamma_d^2(1-f_d)\frac{S_{\varepsilon d}^2}{n_d} + (1-\gamma_d)^2\bar{E}_d^2. \qquad (10)
\end{aligned}$$

The following theorem gives the relation between the design MSE and the model MSE

**Theorem 1** *(i) The design MSE is model unbiased for the model MSE, that is,*

$$E_m\left\{MSE_\pi(\tilde{P}_d^{BLUP})\right\} = MSE_m(\tilde{P}_d^{BLUP}).$$

*(ii) If the fourth-order moments of $\{u_d\}$ and $\{e_{di}\}$ are uniformly bounded, the average across areas of the model MSE is asymptotically equivalent to the average across areas of the design MSE as $D \to \infty$, that is,*

$$\frac{1}{D}\sum_{d=1}^{D} MSE_m(\tilde{P}_d^{BLUP}) - \frac{1}{D}\sum_{d=1}^{D} MSE_\pi(\tilde{P}_d^{BLUP}) \xrightarrow{P} 0.$$

**Proof:** (i) Taking model expectation in (10), we obtain

$$E_m\left\{\text{MSE}_\pi(\tilde{P}_d^{BLUP})\right\} = \gamma_d^2(1-f_d)\frac{E_m(S_{\varepsilon d}^2)}{n_d} + (1-\gamma_d)^2 E_m(\bar{E}_d^2). \tag{11}$$

But note that $\varepsilon_{di} - \bar{E}_d = Y_{di} - P_d + (\mathbf{x}_{di} - \bar{\mathbf{X}}_{di})'\boldsymbol{\beta} = \tilde{Y}_{di} - \tilde{\mathbf{x}}_{di}'\boldsymbol{\beta}$, for $\tilde{Y}_{di} = Y_{di} - P_d$ and $\tilde{\mathbf{x}}_{di} = \mathbf{x}_{di} - \bar{\mathbf{X}}_d$. Thus, $(N_d-1)S_{\varepsilon d}^2 = \sum_{i=1}^{N_d}(\tilde{Y}_{di} - \tilde{\mathbf{x}}_{di}'\boldsymbol{\beta})^2$ is the residual sum of squares of a regression through the origin. Note that it holds

$$\tilde{Y}_{di} - \tilde{\mathbf{x}}_{di}'\boldsymbol{\beta} = e_{di} - N_d^{-1}\sum_{j=1}^{N_d} e_{dj} = \left(1 - \frac{1}{N_d}\right)e_{di} - \frac{1}{N_d}\sum_{j\neq i} e_{dj}.$$

Then, we get for $N_d$ large

$$E_m(\tilde{Y}_{di} - \tilde{\mathbf{x}}_{di}'\boldsymbol{\beta})^2 = \left(1 - \frac{1}{N_d}\right)^2\sigma_e^2 + \frac{1}{N_d^2}(N_d-1)\sigma_e^2 \approx \sigma_e^2.$$

Thus, $E_m(S_{\varepsilon d}^2) \approx \sigma_e^2$ for large $N_d$. Moreover, we have

$$\begin{aligned}
E_m(\bar{E}_d^2) &= E_m\left(u_d + N_d^{-1}\sum_{i=1}^{N_d} e_{di}\right)^2 \\
&= E_m(u_d^2) + E_m\left(N_d^{-1}\sum_{i=1}^{N_d} e_{di}\right)^2 \\
&= \sigma_u^2 + \sigma_e^2/N_d \approx \sigma_u^2.
\end{aligned}$$

Thus, using $f_d = n_d/N_d \approx 0$ and inserting $\gamma_d = \sigma_u^2/(\sigma_u^2 + \sigma_e^2/n_d)$, we have obtained

$$\begin{aligned}
E_m\left\{\text{MSE}_\pi(\tilde{P}_d^{BLUP})\right\} &\approx \gamma_d^2\frac{\sigma_e^2}{n_d} + (1-\gamma_d)^2\sigma_u^2 \\
&= \gamma_d\frac{\sigma_e^2}{n_d} = \text{MSE}_m(\tilde{P}_d^{BLUP}).
\end{aligned}$$

(ii) Using (9) and (10), we get

$$\begin{aligned}
&\frac{1}{D}\sum_{d=1}^{D}\text{MSE}_m(\tilde{P}_d^{BLUP}) - \frac{1}{D}\sum_{d=1}^{D}\text{MSE}_\pi(\tilde{P}_d^{BLUP}) \\
&= \frac{1}{D}\sum_{d=1}^{D}\left\{\gamma_d\frac{\sigma_e^2}{n_d} - \gamma_d^2(1-f_d)\frac{S_{\varepsilon d}^2}{n_d} - (1-\gamma_d)^2\bar{E}_d^2\right\} \\
&= \frac{1}{D}\sum_{d=1}^{D} v_d,
\end{aligned}$$

where $v_d = \gamma_d \frac{\sigma_e^2}{n_d} - \gamma_d^2(1-f_d)\frac{S_{\varepsilon d}^2}{n_d} + (1-\gamma_d)^2 \bar{E}_d^2$. Note that $v_d$ are independent, and for $N_d$ large and for $f_d \approx 0$, their model expectation is

$$
\begin{aligned}
E_m(v_d) &= \gamma_d \frac{\sigma_e^2}{n_d} - \gamma_d^2(1-f_d)\frac{E_m(S_{\varepsilon d}^2)}{n_d} - (1-\gamma_d)^2 E_m(\bar{E}_d^2) \\
&\approx \gamma_d \frac{\sigma_e^2}{n_d} - \gamma_d^2 \frac{\sigma_e^2}{n_d} - (1-\gamma_d)^2 \sigma_v^2 \\
&= (1-\gamma_d)\left\{\gamma_d \frac{\sigma_e^2}{n_d} - (1-\gamma_d)\sigma_v^2\right\} = 0,
\end{aligned}
$$

noting that $\gamma_d \sigma_e^2/n_d = (1-\gamma_d)\sigma_v^2$. Moreover, we have

$$
V_m\left(\frac{1}{D}\sum_{d=1}^{D} v_d\right) = \frac{1}{D^2}\sum_{d=1}^{D} V_m(v_d),
$$

which tends to zero as $D \to \infty$ if $V_m(v_d)$ is bounded. But $V_m(v_d)$ depends on the fourth-order moments of $\{u_d\}$ and $\{e_{di}\}$, which are uniformly bounded. Then $V_m(v_d)$ is bounded and we have showed that $\frac{1}{D}\sum_{d=1}^{D} v_d$ converges to zero in quadratic mean. Since convergence in quadratic mean implies convergence in probability, $\frac{1}{D}\sum_{d=1}^{D} v_d$ converges to zero in probability as $D \to \infty$. □

# Appendix 2: Fitted regression for Structural Survey

Table 1: Model fitting results for LMM

| Variable | Value | Std.Error | DF | t-value | p-value |
|---|---|---|---|---|---|
| (Intercept) | 0.1442500 | 0.0070170 | 285837 | 20.55713 | 0.0000*** |
| Strata1=1 | -0.0070059 | 0.0019959 | 144 | -3.51006 | 0.0006*** |
| District1724=1 | 0.0789174 | 0.0128619 | 144 | 6.13575 | 0.0000*** |
| age∈[16,20) | 0.3487278 | 0.0073290 | 285837 | 47.58162 | 0.0000*** |
| age∈[20,60] | 0.2908208 | 0.0069008 | 285837 | 42.14277 | 0.0000*** |
| age∈[60,64] | 0.1302745 | 0.0076731 | 285837 | 16.97796 | 0.0000*** |
| age=64 | -0.0015075 | 0.0095230 | 285837 | -0.15830 | 0.8742 |
| age≥65 | -0.1158139 | 0.0071840 | 285837 | -16.12112 | 0.0000*** |
| gender=F | -0.0128278 | 0.0094166 | 285837 | -1.36224 | 0.1731 |
| civil status=married | 0.0165010 | 0.0020274 | 285837 | 8.13905 | 0.0000*** |
| civil status=widow/er | 0.0019708 | 0.0057550 | 285837 | 0.34246 | 0.7320 |
| civil status=divorced | 0.0073863 | 0.0031928 | 285837 | 2.31341 | 0.0207* |
| nationality=Swiss | -0.0184364 | 0.0013458 | 285837 | -13.69904 | 0.0000*** |
| secresid=yes | -0.0704905 | 0.0046144 | 285837 | -15.27629 | 0.0000*** |
| housesize=2 | -0.0022829 | 0.0017894 | 285837 | -1.27580 | 0.2020 |
| housesize∈[3,5] | -0.0159658 | 0.0018651 | 285837 | -8.56049 | 0.0000*** |
| housesize∈[6,10] | -0.0326060 | 0.0032409 | 285837 | -10.06066 | 0.0000*** |
| housesize>10 | 0.0131268 | 0.0161541 | 285837 | 0.81260 | 0.4164 |
| Income∈(0,12000] | 0.3519415 | 0.0024847 | 285837 | 141.64245 | 0.0000*** |
| Income∈(12000,24000] | 0.4849786 | 0.0024818 | 285837 | 195.41596 | 0.0000*** |
| Income∈(24000,48000] | 0.5570826 | 0.0020546 | 285837 | 271.14377 | 0.0000*** |
| Income∈(48000,72000] | 0.5771984 | 0.0020013 | 285837 | 288.41058 | 0.0000*** |
| Income∈(72000,96000] | 0.5823435 | 0.0021831 | 285837 | 266.74995 | 0.0000*** |
| Income∈(96000,120000] | 0.5862130 | 0.0021795 | 285837 | 268.97042 | 0.0000*** |
| Income> 120000 | 0.6061222 | 0.0059727 | 285837 | 101.48181 | 0.0000*** |
| OASItri=1 | -0.1852942 | 0.0085840 | 285837 | -21.58584 | 0.0000*** |
| age∈[16,20):gender=F | -0.0137442 | 0.0104941 | 285837 | -1.30971 | 0.1903 |
| age∈[20,60):gender=F | 0.0133350 | 0.0096555 | 285837 | 1.38108 | 0.1673 |
| age∈[60,64):gender=F | -0.0047423 | 0.0106959 | 285837 | -0.44337 | 0.6575 |
| age=64:gender=F | -0.1469525 | 0.0131640 | 285837 | -11.16318 | 0.0000*** |
| age≥ 65:gender=F | 0.0595271 | 0.0100597 | 285837 | 5.91739 | 0.0000*** |
| gender=F:civil status=married | -0.0515040 | 0.0027248 | 285837 | -18.90159 | 0.0000*** |
| gender=F:civil status=widow/er | -0.0440050 | 0.0066260 | 285837 | -6.64127 | 0.0000*** |
| gender=F:civil status=divorced | -0.0130737 | 0.0042418 | 285837 | -3.08211 | 0.0021** |

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

# Appendix 3: Estimates of district proportions

Table 2: Estimated district proportions of active people and estimated percent RRMSE in parenthesis.

| | ZH00 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 |
| size | 1309 | 807 | 3455 | 2113 | 2294 | 3088 | 2600 | 1488 | 3220 | 4075 | 2063 | 9510 |
| GREG | 0.6913 | 0.6719 | 0.7115 | 0.7229 | 0.6801 | 0.6742 | 0.6234 | 0.6861 | 0.6729 | 0.6823 | 0.6782 | 0.7008 |
| LMM | 0.6821 | 0.6803 | 0.6963 | 0.7113 | 0.6783 | 0.6592 | 0.6311 | 0.6838 | 0.6808 | 0.6783 | 0.6703 | 0.6932 |
| %rrmse | (0.847) | (0.962) | (0.617) | (0.713) | (0.737) | (0.634) | (0.747) | (0.82) | (0.633) | (0.575) | (0.736) | (0.426) |
| BM | 0.6873 | 0.6855 | 0.7016 | 0.7168 | 0.6835 | 0.6642 | 0.6359 | 0.6890 | 0.6860 | 0.6835 | 0.6755 | 0.6985 |
| %rrmse | (1.118) | (1.211) | (0.927) | (0.962) | (1.05) | (0.98) | (1.046) | (1.115) | (0.999) | (0.92) | (1.068) | (0.843) |
| | BE02 | BE00 | | | | | | | | | LU00 | |
| Nr | 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 301 | 302 |
| size | 2706 | 2420 | 1800 | 2079 | 2556 | 10294 | 2870 | 404 | 1013 | 1210 | 909 | 3493 |
| GREG | 0.6219 | 0.6234 | 0.7136 | 0.6601 | 0.6815 | 0.6673 | 0.6409 | 0.7061 | 0.6285 | 0.6608 | 0.667 | 0.6952 |
| LMM | 0.6216 | 0.6177 | 0.6925 | 0.6556 | 0.6672 | 0.6621 | 0.6433 | 0.6474 | 0.6365 | 0.6514 | 0.6431 | 0.6819 |
| %rrmse | (0.703) | (0.755) | (0.815) | (0.751) | (0.724) | (0.4) | (0.724) | (1.111) | (0.999) | (0.888) | (0.927) | (0.59) |
| BM | 0.6263 | 0.6224 | 0.6978 | 0.6606 | 0.6723 | 0.6672 | 0.6482 | 0.6524 | 0.6414 | 0.6564 | 0.6480 | 0.6870 |
| %rrmse | (0.995) | (1.069) | (1.074) | (1.054) | (1.01) | (0.824) | (1.056) | (1.349) | (1.234) | (1.186) | (1.194) | (0.928) |
| | LU00 | | | UR00 | SZ00 | | | | | | OW00 | NW00 |
| Nr | 303 | 304 | 305 | 400 | 501 | 502 | 503 | 504 | 505 | 506 | 600 | 700 |
| size | 9168 | 3634 | 2503 | 895 | 371 | 62 | 703 | 333 | 1073 | 1405 | 922 | 1183 |
| GREG | 0.6647 | 0.7 | 0.6866 | 0.6277 | 0.6531 | 0.6125 | 0.7031 | 0.7109 | 0.6984 | 0.6831 | 0.7212 | 0.6625 |
| LMM | 0.6626 | 0.7004 | 0.6914 | 0.6365 | 0.6793 | 0.6354 | 0.6889 | 0.6929 | 0.6973 | 0.6815 | 0.6977 | 0.6696 |
| %rrmse | (0.402) | (0.616) | (0.659) | (0.991) | (1.215) | (1.741) | (1.091) | (1.199) | (0.917) | (0.865) | (0.939) | (0.888) |
| BM | 0.6676 | 0.7058 | 0.6967 | 0.6413 | 0.6845 | 0.6403 | 0.6941 | 0.6982 | 0.7027 | 0.6867 | 0.7030 | 0.6747 |
| %rrmse | (0.824) | (0.938) | (0.947) | (1.208) | (1.417) | (1.939) | (1.306) | (1.374) | (1.175) | (1.119) | (1.186) | (1.208) |
| | GL00 | ZG00 | FR00 | | | | | | | SO00 | | |
| Nr | 800 | 900 | 1001 | 1002 | 1003 | 1004 | 1005 | 1006 | 1007 | 1101 | 1102 | 1103 |
| size | 1030 | 6086 | 730 | 571 | 1238 | 2509 | 816 | 1050 | 448 | 493 | 347 | 221 |
| GREG | 0.6852 | 0.6833 | 0.6723 | 0.6756 | 0.6492 | 0.678 | 0.688 | 0.6981 | 0.7263 | 0.6632 | 0.6505 | 0.6715 |
| LMM | 0.6682 | 0.6798 | 0.6756 | 0.6752 | 0.6665 | 0.6701 | 0.6865 | 0.6790 | 0.7076 | 0.6901 | 0.6500 | 0.6784 |
| %rrmse | (0.918) | (0.467) | (1.031) | (1.135) | (0.879) | (0.74) | (0.959) | (0.953) | (0.972) | (1.044) | (1.208) | (1.255) |
| BM | 0.6733 | 0.6850 | 0.6807 | 0.6803 | 0.6715 | 0.6752 | 0.6917 | 0.6842 | 0.713 | 0.6953 | 0.6549 | 0.6835 |
| %rrmse | (1.165) | (0.908) | (1.265) | (1.327) | (1.153) | (1.066) | (1.12) | (1.152) | (1.165) | (1.259) | (1.431) | (1.44) |
| | SO00 | | | | | | | BS00 | BL00 | | | |
| Nr | 1104 | 1105 | 1106 | 1107 | 1108 | 1109 | 1110 | 1200 | 1301 | 1302 | 1303 | 1304 |
| size | 523 | 619 | 1261 | 1195 | 1396 | 428 | 339 | 4609 | 4070 | 509 | 1431 | 873 |
| GREG | 0.6169 | 0.6403 | 0.6641 | 0.6425 | 0.6554 | 0.6908 | 0.5957 | 0.6075 | 0.6015 | 0.668 | 0.6536 | 0.676 |
| LMM | 0.6369 | 0.6519 | 0.6568 | 0.6357 | 0.6587 | 0.6851 | 0.6311 | 0.6001 | 0.5979 | 0.6579 | 0.6375 | 0.6720 |
| %rrmse | (1.235) | (1.055) | (0.933) | (0.874) | (0.969) | (1.136) | (1.227) | (0.601) | (0.676) | (1.138) | (0.943) | (0.926) |
| BM | 0.6417 | 0.6568 | 0.6618 | 0.6405 | 0.6637 | 0.6903 | 0.6359 | 0.6047 | 0.6024 | 0.6629 | 0.6424 | 0.6771 |
| %rrmse | (1.447) | (1.29) | (1.17) | (1.121) | (1.222) | (1.254) | (1.384) | (0.964) | (1.042) | (1.314) | (1.148) | (1.139) |
| | BL00 | SH00 | | | | | | AR00 | | | AI00 | SG00 |

| Nr | 1305 | 1401 | 1402 | 1403 | 1404 | 1405 | 1406 | 1501 | 1502 | 1503 | 1600 | 1721 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| size | 399 | 106 | 202 | 1368 | 90 | 162 | 122 | 600 | 413 | 389 | 362 | 3051 |
| GREG | 0.6548 | 0.7135 | 0.741 | 0.6253 | 0.6717 | 0.6936 | 0.7448 | 0.6773 | 0.6734 | 0.7078 | 0.686 | 0.6634 |
| LMM | 0.6501 | 0.6798 | 0.6689 | 0.6246 | 0.6244 | 0.629 | 0.6551 | 0.6669 | 0.6584 | 0.6648 | 0.6701 | 0.6583 |
| %rrmse | (1.268) | (1.335) | (1.237) | (0.879) | (1.569) | (1.37) | (1.489) | (1.074) | (1.062) | (1.176) | (1.004) | (0.677) |
| BM | 0.655 | 0.685 | 0.674 | 0.6294 | 0.6292 | 0.6338 | 0.6601 | 0.672 | 0.6635 | 0.6699 | 0.6752 | 0.6633 |
| %rrmse | (1.435) | (1.516) | (1.439) | (1.161) | (1.693) | (1.526) | (1.631) | (1.295) | (1.321) | (1.357) | (1.266) | (1.022) |

| | SG00 | | | | | | | GR00 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 1722 | 1723 | 1724 | 1725 | 1726 | 1727 | 1728 | 1821 | 1822 | 1823 | 1824 | 1825 |
| size | 1039 | 1767 | 891 | 966 | 1678 | 1136 | 1841 | 256 | 120 | 326 | 477 | 261 |
| GREG | 0.6506 | 0.6845 | 0.703 | 0.693 | 0.6658 | 0.6541 | 0.6701 | 0.6486 | 0.576 | 0.6397 | 0.6507 | 0.6686 |
| LMM | 0.6604 | 0.6723 | 0.6785 | 0.6527 | 0.6703 | 0.6459 | 0.6694 | 0.6431 | 0.5776 | 0.6411 | 0.6728 | 0.6560 |
| %rrmse | (0.953) | (0.789) | (1.413) | (0.925) | (0.848) | (0.913) | (0.745) | (1.185) | (1.57) | (1.26) | (1.116) | (1.334) |
| BM | 0.6654 | 0.6774 | 0.6837 | 0.6577 | 0.6754 | 0.6508 | 0.6745 | 0.6480 | 0.5821 | 0.6459 | 0.678 | 0.6610 |
| %rrmse | (1.166) | (1.061) | (1.491) | (1.188) | (1.094) | (1.196) | (1.064) | (1.395) | (1.707) | (1.552) | (1.332) | (1.517) |

| | GR00 | | | | | | AG00 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 1826 | 1827 | 1828 | 1829 | 1830 | 1831 | 1901 | 1902 | 1903 | 1904 | 1905 | 1906 |
| size | 690 | 493 | 214 | 1096 | 660 | 541 | 3805 | 6956 | 3683 | 2409 | 1954 | 1594 |
| GREG | 0.7163 | 0.6892 | 0.6042 | 0.6746 | 0.6615 | 0.6071 | 0.6723 | 0.6944 | 0.6692 | 0.6629 | 0.6607 | 0.6709 |
| LMM | 0.6845 | 0.6871 | 0.5966 | 0.6584 | 0.6604 | 0.6294 | 0.6689 | 0.6806 | 0.6765 | 0.6615 | 0.6512 | 0.6734 |
| %rrmse | (1.029) | (1.062) | (1.379) | (0.924) | (1.058) | (1.124) | (0.608) | (0.433) | (0.584) | (0.653) | (0.758) | (0.807) |
| BM | 0.6897 | 0.6923 | 0.6011 | 0.6634 | 0.6654 | 0.6342 | 0.674 | 0.6857 | 0.6816 | 0.6666 | 0.6562 | 0.6785 |
| %rrmse | (1.285) | (1.237) | (1.528) | (1.197) | (1.276) | (1.376) | (0.963) | (0.857) | (0.943) | (1.008) | (0.995) | (1.075) |

| | AG00 | | | | | TG00 | | | | | TI00 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 1907 | 1908 | 1909 | 1910 | 1911 | 2011 | 2012 | 2013 | 2014 | 2015 | 2101 | 2102 |
| size | 2941 | 1575 | 2299 | 3367 | 1645 | 2656 | 3324 | 2316 | 2276 | 2603 | 2577 | 262 |
| GREG | 0.6883 | 0.7298 | 0.6661 | 0.6794 | 0.6564 | 0.6378 | 0.6897 | 0.6794 | 0.6971 | 0.6989 | 0.5797 | 0.5559 |
| LMM | 0.6833 | 0.7177 | 0.6569 | 0.6803 | 0.6562 | 0.6404 | 0.6856 | 0.6670 | 0.6971 | 0.6902 | 0.5812 | 0.5390 |
| %rrmse | (0.632) | (0.79) | (0.718) | (0.587) | (0.823) | (0.658) | (0.633) | (0.738) | (0.665) | (0.652) | (0.804) | (1.522) |
| BM | 0.6885 | 0.7232 | 0.6619 | 0.6855 | 0.6612 | 0.6453 | 0.6908 | 0.6721 | 0.7024 | 0.6954 | 0.5856 | 0.5431 |
| %rrmse | (0.964) | (1.023) | (0.952) | (0.901) | (1.086) | (0.943) | (0.996) | (1.01) | (0.967) | (0.964) | (1.053) | (1.630) |

| | TI00 | | | | | | VD00 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 2103 | 2104 | 2105 | 2106 | 2107 | 2108 | 2221 | 2222 | 2223 | 2224 | 2225 | 2226 |
| size | 512 | 3271 | 7664 | 2739 | 685 | 314 | 1986 | 1923 | 2029 | 4325 | 7617 | 2915 |
| GREG | 0.57 | 0.5601 | 0.5675 | 0.5535 | 0.5774 | 0.5962 | 0.6083 | 0.6531 | 0.7063 | 0.6441 | 0.6509 | 0.613 |
| LMM | 0.5810 | 0.5579 | 0.5731 | 0.5578 | 0.5733 | 0.5828 | 0.6184 | 0.6398 | 0.6930 | 0.6343 | 0.6473 | 0.6147 |
| %rrmse | (1.23) | (0.824) | (0.524) | (0.869) | (1.238) | (1.468) | (0.828) | (0.789) | (0.692) | (0.58) | (0.457) | (0.744) |
| BM | 0.5854 | 0.5622 | 0.5774 | 0.562 | 0.5777 | 0.5873 | 0.6231 | 0.6447 | 0.6983 | 0.6391 | 0.6523 | 0.6194 |
| %rrmse | (1.451) | (1.128) | (0.885) | (1.112) | (1.444) | (1.611) | (1.081) | (1.093) | (1.017) | (0.908) | (0.893) | (1.022) |

| | VD00 | | | | VS00 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 2227 | 2228 | 2229 | 2230 | 2301 | 2302 | 2303 | 2304 | 2305 | 2306 | 2307 | 2308 |
| size | 3861 | 4498 | 3468 | 3993 | 661 | 625 | 371 | 138 | 301 | 297 | 1152 | 1110 |
| GREG | 0.6603 | 0.6492 | 0.6543 | 0.6249 | 0.6049 | 0.663 | 0.6294 | 0.5769 | 0.6146 | 0.6043 | 0.6461 | 0.6459 |
| LMM | 0.6562 | 0.6469 | 0.6468 | 0.6142 | 0.6306 | 0.6521 | 0.6232 | 0.6076 | 0.5931 | 0.6261 | 0.6548 | 0.6465 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| %rrmse | (0.633) | (0.547) | (0.582) | (0.669) | (1.088) | (1.086) | (1.295) | (1.41) | (1.422) | (1.282) | (0.936) | (0.926) |
| BM | 0.6612 | 0.6518 | 0.6517 | 0.6189 | 0.6354 | 0.6571 | 0.6279 | 0.6122 | 0.5976 | 0.6309 | 0.6597 | 0.6515 |
| %rrmse | (0.864) | (0.899) | (0.961) | (0.997) | (1.383) | (1.254) | (1.461) | (1.57) | (1.564) | (1.438) | (1.208) | (1.157) |

| | VS00 | | | | | NE00 | | | | | | GE00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr | 2309 | 2310 | 2311 | 2312 | 2313 | 2401 | 2402 | 2403 | 2404 | 2405 | 2406 | 2500 |
| size | 287 | 355 | 1177 | 1124 | 663 | 2052 | 1980 | 696 | 2665 | 814 | 581 | 20786 |
| GREG | 0.6144 | 0.6621 | 0.6161 | 0.6325 | 0.6723 | 0.6304 | 0.6358 | 0.593 | 0.6347 | 0.6821 | 0.6044 | 0.6218 |
| LMM | 0.6211 | 0.6462 | 0.6176 | 0.6324 | 0.6743 | 0.6239 | 0.6223 | 0.6109 | 0.6293 | 0.6710 | 0.5865 | 0.6124 |
| %rrmse | (1.33) | (1.177) | (0.949) | (0.924) | (0.994) | (0.788) | (0.801) | (1.072) | (0.695) | (0.905) | (1.145) | (0.31) |
| BM | 0.6259 | 0.6511 | 0.6223 | 0.6373 | 0.6795 | 0.6287 | 0.6270 | 0.6156 | 0.6341 | 0.6761 | 0.5910 | 0.6171 |
| %rrmse | (1.457) | (1.38) | (1.227) | (1.17) | (1.164) | (1.084) | (1.087) | (1.303) | (1.024) | (1.134) | (1.315) | (0.81) |

| | JU00 | | |
|---|---|---|---|
| Nr | 2601 | 2602 | 2603 |
| size | 1817 | 493 | 1292 |
| GREG | 0.6064 | 0.6858 | 0.5935 |
| LMM | 0.6147 | 0.6448 | 0.5813 |
| %rrmse | (0.889) | (1.125) | (0.975) |
| BM | 0.6194 | 0.6497 | 0.5857 |
| %rrmse | (1.118) | (1.361) | (1.211) |